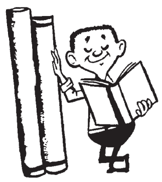
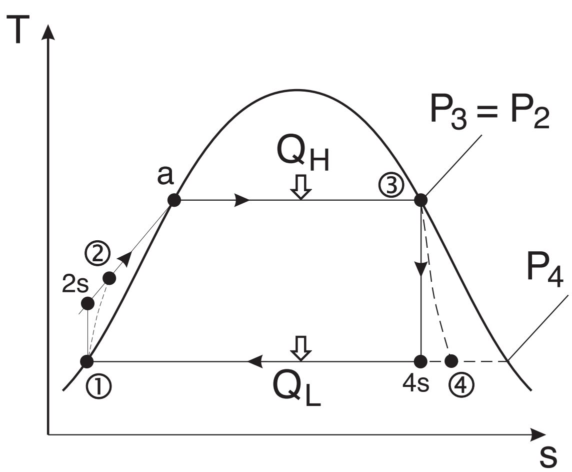
Rankine Cycle

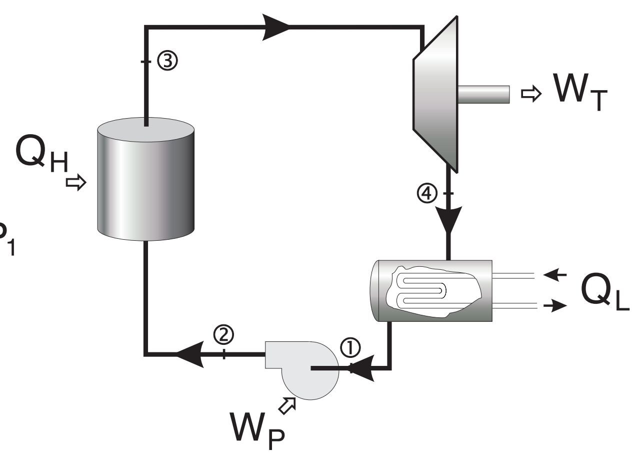


Reading	Problems
9-2 ⟶ 9-7	9-16, 9-23, 9-30, 9-38, 9-43, 9-84

Definitions

- working fluid is alternately vaporized and condensed as it recirculates in a closed cycle
- water is typically used as the working fluid because of its low cost and relatively large value of enthalpy of vaporization
- the standard vapour cycle that excludes internal irreversibilities is called the **Ideal Rankine Cycle**





- the condensation process is allowed to proceed to completion between state points $\mathbf{4} \longrightarrow \mathbf{1}$
  - provides a saturated liquid at 1
- the water at state point 1 can be conveniently pumped to the boiler pressure at state point 2
- but the water is <u>not</u> at the saturation temperature corresponding to the boiler pressure
- heat must be added to change the water at 2 to saturated water at 'a'
- when heat is added at non-constant temperature (between $\boldsymbol{2 - a}$), the cycle efficiency will decrease

Analyze the Process

Assume steady flow, $KE = PE = 0$.

From a 1st law balance, we know

$$energy\ in = energy\ out$$

Device	1st Law Balance			
Boiler	$h_2 + q_H = h_3$	$\Rightarrow$	$q_H = h_3 - h_2$	(in)
Turbine	$h_3 = h_4 + w_T$	$\Rightarrow$	$w_T = h_3 - h_4$	(out)
Condenser	$h_4 = h_1 + q_L$	$\Rightarrow$	$q_L = h_4 - h_1$	(out)
Pump	$h_1 + w_P = h_2$	$\Rightarrow$	$w_P = h_2 - h_1$	(in)

The net work output is given as

$$\begin{aligned} w_T - w_p &= (h_3 - h_4) - (h_2 - h_1) \\ &= (h_3 - h_4) + (h_1 - h_2) \end{aligned}$$

The net heat supplied to the boiler is

$$q_H = (h_3 - h_2)$$

The Rankine efficiency is

$$\begin{aligned} \eta_R &= \frac{net\ work\ output}{heat\ supplied\ to\ the\ boiler} \\ &= \frac{(h_3 - h_4) + (h_1 - h_2)}{(h_3 - h_2)} \end{aligned}$$

If we consider the fluid to be incompressible

$$(h_2 - h_1) = v(P_2 - P_1)$$

Since the actual process is irreversible, an isentropic efficiency can be defined such that

Expansion process $\Rightarrow$ Isentropic efficiency $= \dfrac{\text{actual work}}{\text{isentropic work}}$

Compression process $\Rightarrow$ Isentropic efficiency $= \dfrac{\text{isentropic work}}{\text{actual work}}$

Both isentropic efficiencies will have a numerical value between 0 and 1.

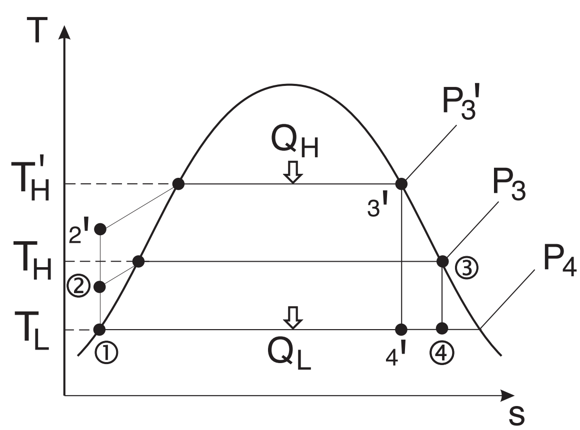
Effects of Boiler and Condenser Pressure

We know the efficiency is proportional to

$$\eta \propto 1 - \frac{T_L}{T_H}$$

The question is $\longrightarrow$ how do we increase efficiency $\Rightarrow$ $T_L \downarrow$ and/or $T_H \uparrow$.

1. INCREASED BOILER PRESSURE:



- an increase in boiler pressure results in a higher T_H for the same T_L, therefore $\eta \uparrow$.
- but $4'$ has a lower quality than 4
  - wetter steam at the turbine exhaust

- results in cavitation of the turbine blades
- $\eta \downarrow$ plus $\uparrow$ maintenance

- quality should be $> 90\%$ at the turbine exhaust

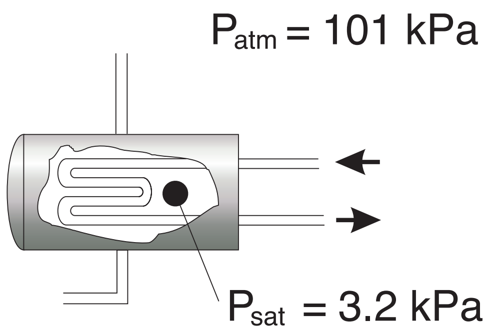
2. LOWER T_L:

- we are generally limited by the TER (lake, river, etc.)

  eg. lake @ $15\,°C + \underbrace{\Delta T = 10\,°C}_{resistance\ to\ HT} = 25\,°C$

  $\Rightarrow\ P_{sat} = 3.2\ kPa.$

- this is why we have a condenser
  - the pressure at the exit of the turbine can be less than atmospheric pressure
  - the closed loop of the condenser allows us to use treated water on the cycle side
  - but if the pressure is less that atmospheric pressure, air can leak into the condenser, preventing condensation



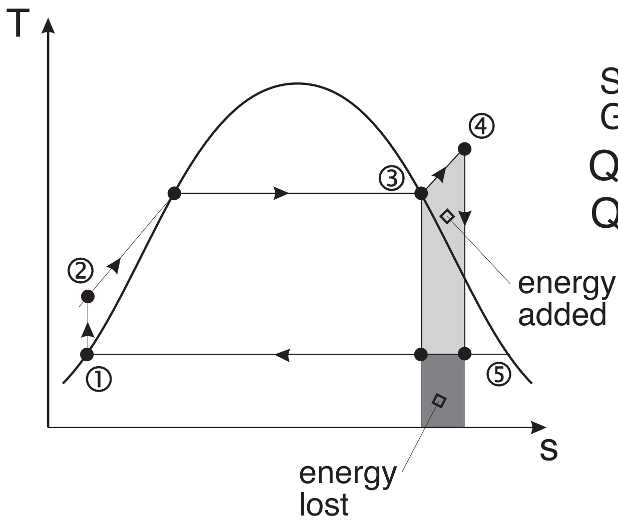
3. INCREASED T_H BY ADDING SUPERHEAT:

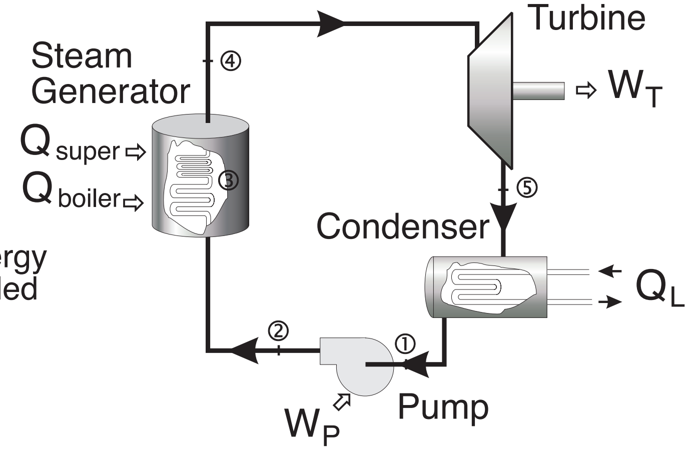
- the average temperature at which heat is supplied in the boiler can be increased by superheating the steam
  - dry saturated steam from the boiler is passed through a second bank of smaller bore tubes within the boiler until the steam reaches the required temperature

    The advantage is

    $$\eta = \frac{W_{net} \uparrow}{Q_H \uparrow} \quad overall \uparrow$$

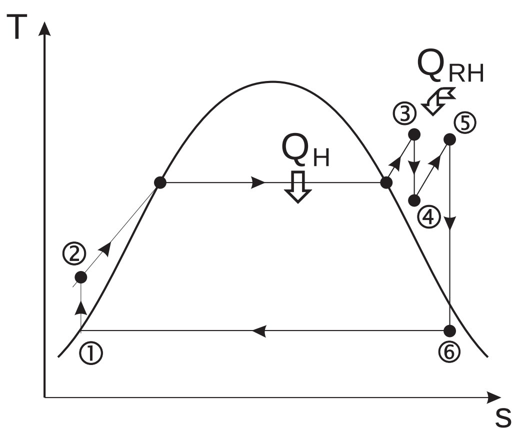
    The value of $\overline{T}_H$, the mean temperature at which heat is added, increases, while $\overline{T}_L$ remains constant. Therefore the efficiency increases.
  - the quality of the turbine exhaust increases, hopefully where $x > 0.9$
  - with sufficient superheating the turbine exhaust can fall in the superheated region.

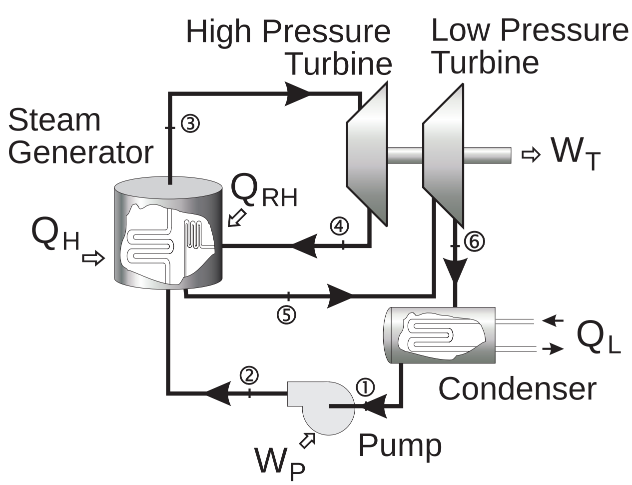




Rankine Cycle with Reheat

- the wetness at the exhaust of the turbine should be no greater that 10% - this can result in physical erosion of the turbine blades
- but high boiler pressures are required for high efficiency - tends to lead to a high wetness ratio
- to improve the exhaust steam conditions, the steam can be reheated with the expansion carried out in two steps

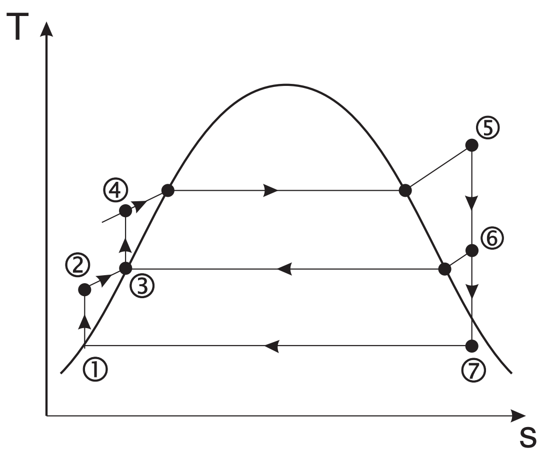


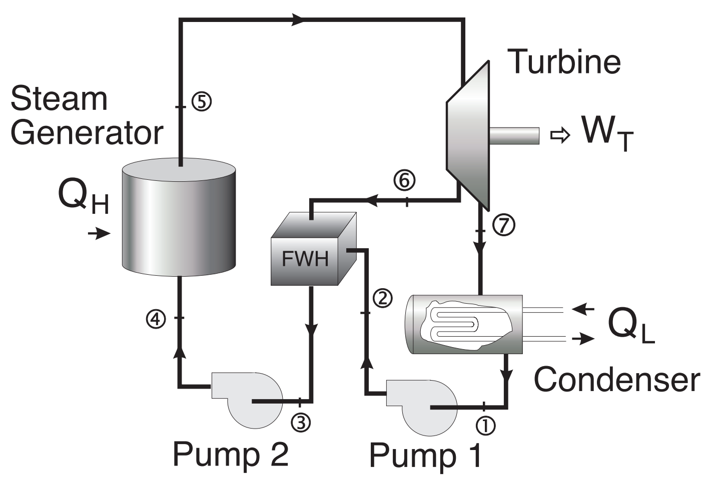


- the temperature of the steam entering the turbine is limited by metallurgical constraints
- modern boilers can handle up to $\mathbf{30}\ \boldsymbol{MPa}$ and a maximum temperature of $\boldsymbol{T_{max} \approx 650\ ^\circ C}$.
- newer materials, such as ceramic blades can handle temperatures up to $\mathbf{750\ ^\circ} \boldsymbol{C}$.

Rankine Cycle with Regeneration

- Carnot cycle has efficiency: $\eta = 1 - T_L/T_H$
  - add Q_H at as high a T_H as possible
  - reject Q_L at as low a T_L as possible
- the Rankine cycle can be used with a ***Feedwater Heater*** to heat the high pressure sub-cooled water at the pump exit to the saturation temperature
  - most of the heat addition (Q_H) is done at high temperature





Feedwater Heaters

There are two different types of feedwater heaters

1. ***OPEN FWH:*** the streams mix $\longrightarrow$ high temperature steam with low temperature water at constant pressure
2. ***CLOSED FWH:*** a heat exchanger is used to transfer heat between the two streams but the streams do *not* mix. The two streams can be maintained at different pressures.

1. ***OPEN FWH:***

   - working fluid passes isentropically through the turbine stages and pumps
   - steam enters the first stage turbine at state 1 and expands to state 2 - where a fraction of the total flow is bled off into an open feedwater heater at P_2
   - the rest of the steam expands into the second stage turbine at state point 3 - this portion of the fluid is condensed and pumped as a saturated liquid to the FWH at P_2
   - a single mixed stream exists the FWH at state point 6

Analysis:

- we must determine the mass flow rates through each of the components.

  By performing an mass balance over the turbine

  $$\dot{m}_2 + \dot{m}_3 = \dot{m}_1 \qquad (1)$$

  If we normalize Eq. (1) with respect the total mass flow rate $\dot{m}_1$

  $$\frac{\dot{m}_2}{\dot{m}_1} + \frac{\dot{m}_3}{\dot{m}_1} = 1 \qquad (2)$$

  Let the flow at state point 2 be

  $$y = \frac{\dot{m}_2}{\dot{m}_1}$$

  Therefore

  $$\frac{\dot{m}_3}{\dot{m}_1} = 1 - y \qquad (3)$$

  Assuming no heat loss at the FWH, establish an energy balance across the FWH

  $$y h_2 + (1 - y) h_5 = 1 \cdot h_6$$

  $$\boxed{y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{\dot{m}_2}{\dot{m}_1}}$$

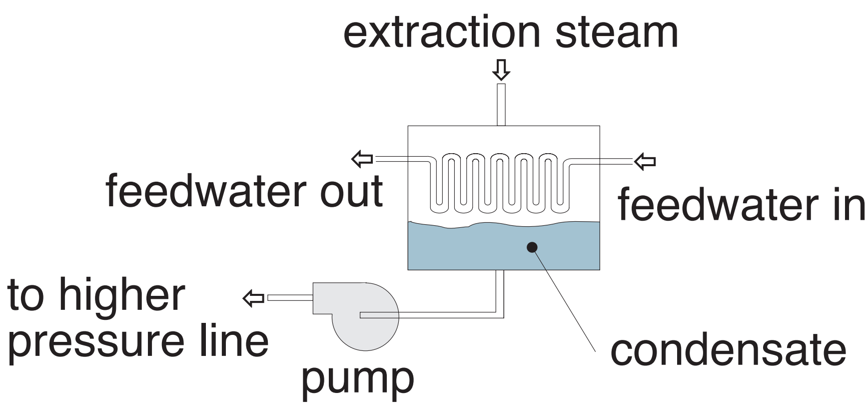
  and

  $$\boxed{1 - y = \frac{\dot{m}_3}{\dot{m}_1}}$$

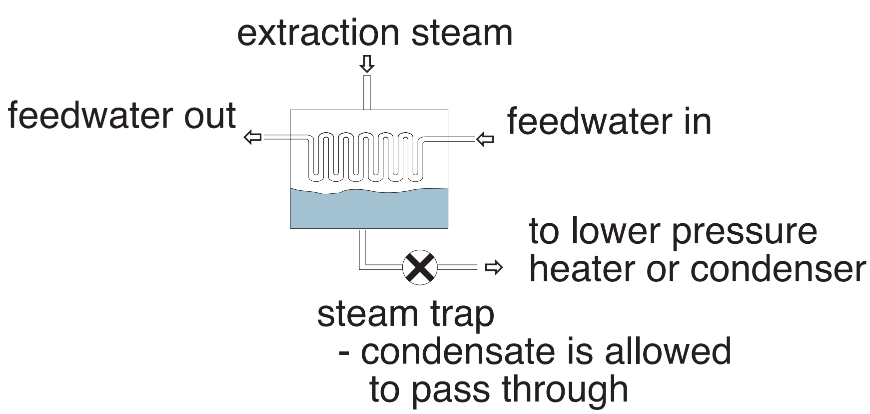
2. ***CLOSED FWH:***

- two variations exist

  (a) pump the condensate back to the high pressure line



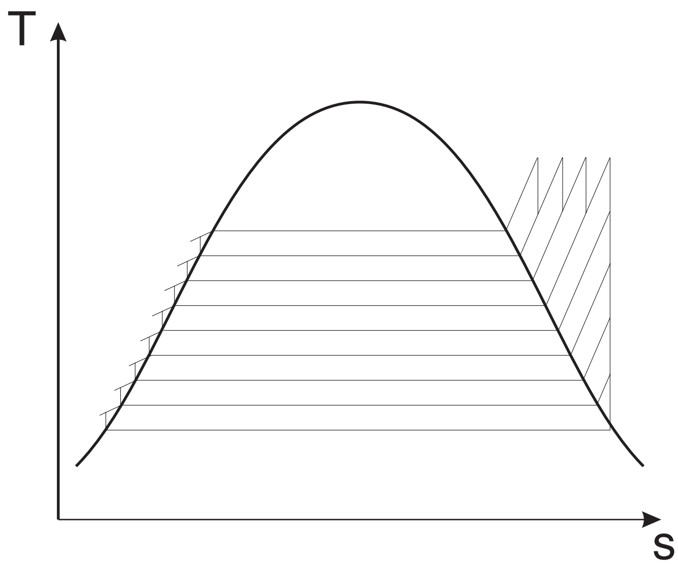
  (b) – a steam trap is inserted in the condensed steam line that allows only liquid to pass
  – liquid is passed to a low pressure region such as the condenser or a low pressure heater



- the incoming feedwater does not mix with the extracted steam
  - both streams flow separately through the heater
  - the two streams can have different pressures

Other Topics

"IDEAL" RANKINE CYCLE:



- too expensive to build
- requires multiple reheat and regeneration cycles
- approaches Carnot efficiency

TOPPING CYCLE (BINARY CYCLE):

- involves two Rankine cycles running in tandem with different working fluids such as mercury and water
- why:
  - typically a boiler will supply energy at $1300 - 1400\ °C$
  - but $T_{critical}$ for water is $374.14\ °C$
    * most energy is absorbed below this temperature
    * high ΔT between the boiler source and the water leads to a major source of irreversibilities
  - $T_{critical}$ for mercury is about $1500\ °C$
    * no need for superheating
  - combine the large enthalpy of evaporation of water at low temperatures with the advantages of mercury at high temperatures
  - in addition, the mercury dome leads to a high quality at the exit of the turbine