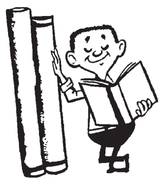
Availability



Reading	**Problems**
8.1 → 8.8	8-33, 8-36, 8-43, 8-53, 8-65, 8-76, 8-94, 8-105, 8-121, 8-140

Second Law Analysis of Systems

AVAILABILITY:

- the theoretical maximum amount of work that can be obtained from a system at a given state P_1 and T_1 when interacting with a reference atmosphere at the constant pressure and temperature P_0 and T_0.
- also referred to as "exergy".

The following observations can be made about availability:

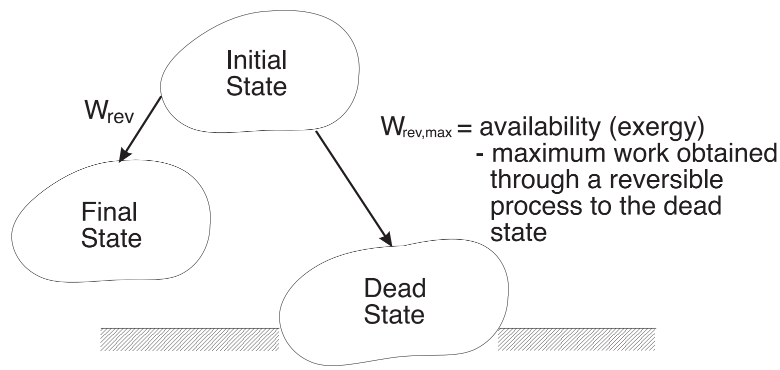
1. Availability is a **property** - since any quantity that is fixed when the state is fixed is a property.
2. Availability is a **composite property** - since its value depends upon an external datum - the temperature and pressure of the dead state.
3. Availability of a system is 0 at its **dead state** when $T = T_0$ and $P = P_0$.
4. Unless otherwise stated, assume the dead state to be:

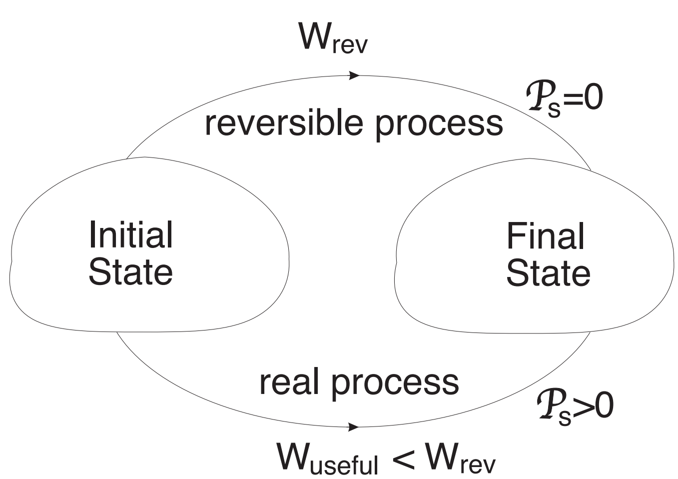
$$P_0 = 1\,atm$$

$$T_0 = 25^\circ C$$

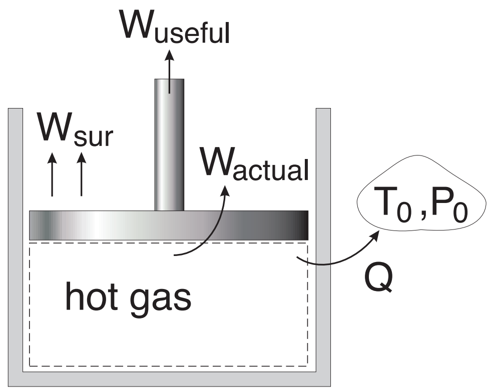
5. The maximum work is obtained through a reversible process to the dead state.

$$\underbrace{REVERSIBLE\;WORK}_{W_{rev}} = \underbrace{USEFUL\;WORK}_{W_{useful}} + \underbrace{IRREVERSIBILITY}_{X_{des}}$$


Initial
State
W rev
Final
State
W rev,max = availability (exergy)
- maximum work obtained
through a reversible
process to the dead
state
Dead
State


W rev
reversible process
𝒫s=0
Initial
State
Final
State
real process
𝒫s>0
W useful < W rev

Control Mass Analysis



- we know

$$W_{rev} = W_{useful} \; + \; X_{des}$$

but as shown in the figure, the actual work of the process is divided into two components

$$W_{actual} = W_{useful} + W_{sur}$$

- where W_{sur} is the part of the work done against the surroundings to displace the ambient air

$$W_{sur} = P_0(V_2 - V_1) = -P_0(V_1 - V_2)$$

- this is unavoidable $\longrightarrow$ this is not useful work. Nothing is gained by pushing the atmosphere away.

To find W_{actual}, from the 1st law

$$E_1 - Q - W_{actual} = E_2 \quad \longrightarrow \quad Q = E_1 - E_2 - W_{actual}$$

From the 2nd law

$$\begin{aligned} S_{gen} &= \Delta S_{system} + \Delta S_{sur} \geq 0 \\ &= S_2 - S_1 + \frac{Q}{T_0} \end{aligned}$$

But from the 1st law balance we know

$$\frac{Q}{T_0} = \frac{E_1 - E_2 - W_{actual}}{T_0}$$

and when we combine this with the 2nd law

$$S_{gen} = S_2 - S_1 + \frac{E_1 - E_2 - W_{actual}}{T_0}$$

which leads to

$$W_{actual} = (E_1 - E_2) + T_0(S_2 - S_1) - T_0 S_{gen}$$

or by reversing the order of S_2 and S_1

$$W_{actual} = (E_1 - E_2) - T_0(S_1 - S_2) - T_0 S_{gen}$$

But we also know that

$$W_{useful} = W_{actual} - W_{sur}$$

therefore

$$W_{useful} = (E_1 - E_2) - T_0(S_1 - S_2) + \underbrace{P_0(V_1 - V_2)}_{-W_{sur}} - T_0 S_{gen}$$

and

$$\begin{aligned} W_{rev} &= W_{useful} + X_{des} \\ &= W_{actual} - W_{sur} + X_{des} \end{aligned}$$

where

$$X_{des} = T_0 S_{gen}$$

Therefore

$$W_{rev} = (E_1 - E_2) - T_0(S_1 - S_2) + P_0(V_1 - V_2)$$

In summary

$$\boxed{\begin{aligned} W_{actual} &= (E_1 - E_2) - T_0(S_1 - S_2) - T_0 S_{gen} \\ W_{useful} &= (E_1 - E_2) - T_0(S_1 - S_2) + P_0(V_1 - V_2) - T_0 S_{gen} \\ X = W_{rev} &= (E_1 - E_2) - T_0(S_1 - S_2) + P_0(V_1 - V_2) \end{aligned}}$$

Define

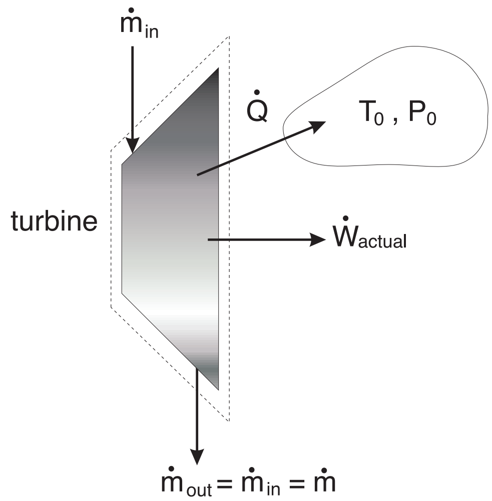
$$\begin{aligned} X &= CONTROL\ MASS\ AVAILABILITY \\ &= W_{rev} \text{ (in going to the dead state)} \\ &= (E - E_0) - T_0(S - S_0) + P_0(V - V_0) \end{aligned}$$

where the specific availability is defined as

$$\phi = \frac{X}{m}$$

Control Volume Analysis

Consider a steady state, steady flow (SS-SF) process



From the 1st law

$$\frac{dE_{cv}}{dt}\nearrow^{0} = -\dot{W}_{actual} - \dot{Q} + \left[\dot{m}(h + \frac{(v^*)^2}{2} + gz)\right]_{in} - \left[\dot{m}(h + \frac{(v^*)^2}{2} + gz)\right]_{out} \qquad (1)$$

From the 2nd law

$$\frac{dS_{cv}}{dt}\nearrow^{0} = \left(\dot{m}s + \frac{\dot{Q}\nearrow^{0}}{T_{TER}}\right)_{in} - \left(\dot{m}s + \frac{\dot{Q}}{T_0}\right)_{out} + \dot{S}_{gen} \qquad (2)$$

Combining (1) and (2) through the $\dot{Q}$ term, leads to the actual work output of the turbine, given as

$$\begin{aligned} \dot{W}_{actual} &= \left[\dot{m}\left(h + \frac{(v^*)^2}{2} + gz - T_0 s\right)\right]_{in} - \left[\dot{m}\left(h + \frac{(v^*)^2}{2} + gz - T_0 s\right)\right]_{out} - T_0\dot{S}_{gen} \\ &= \dot{m}\left[-T_0\Delta s + \Delta h + \Delta KE + \Delta PE\right] - (T_0\dot{S}_{gen}) \qquad (3) \end{aligned}$$

The specific flow availability, ψ, is given as

$$\psi = -T_0(s - s_0) + (h - h_0) + \left(\frac{(v^*)^2}{2} - \frac{(v_0^*\nearrow^{0})^2}{2}\right) + g(z - z_0\nearrow^{0}) \qquad (4)$$

Efficiency and Effectiveness

1. **First law efficiency (thermal efficiency)**

$$\eta = \frac{net\ work\ output}{gross\ heat\ input} = \frac{W_{net}}{Q_{in}}$$

Carnot cycle

$$\eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

2. **Second Law Efficiency (effectiveness)**

$$\eta_{2nd} = \frac{net\ work\ output}{maximum\ reversible\ work} = \frac{net\ work\ output}{availability}$$

Turbine $\rightarrow$ $\eta_{2nd} = \frac{\dot{W}/\dot{m}}{\psi_e - \psi_i}$

Compressor $\rightarrow$ $\eta_{2nd} = \frac{\psi_e - \psi_i}{\dot{W}/\dot{m}}$

Heat Source $\rightarrow$ $\eta_{2nd} = \frac{\dot{W}/\dot{m}}{\dot{Q}/\dot{m}\left[1 - \frac{T_0}{T_{TER}}\right]}$

3. **Isentropic efficiency (process efficiency)**

   (a) adiabatic turbine efficiency

$$\eta_T = \frac{work\ of\ actual\ adiabatic\ expansion}{work\ of\ reversible\ adiabatic\ expansion} = \frac{W_{act}}{W_S}$$

   (b) adiabatic compressor efficiency

$$\eta_C = \frac{work\ of\ reversible\ adiabatic\ compression}{work\ of\ actual\ adiabatic\ compression} = \frac{W_S}{W_{act}}$$