Week 2: Lecture 3

Incompressible Liquids

- a substance whose volume cannot be changed
- no substance is truly incompressible, but this model is good for most liquids and solids

A simple compressible substance has only one work mode and that work mode is Pdv work.

If a substance is assumed to be incompressible then its internal energy, for example, cannot be varied independently by work transfer but it can be varied by heat transfer at constant volume.

For a incompressible substance

$$c_p = c_v = c$$

From Gibb's equation we can see that

$$s_2 - s_1 = c \ln \left(\frac{T_2}{T_1} \right)$$

For an isothermal process ($T_2 = T_1$) we see that the process becomes isentropic ($S_2 - S_1$).

In summary

$$\begin{aligned} u_2 - u_1 &= c(T_2 - T_1) \\ h_2 - h_1 &= (u_2 - u_1) + v(P_2 - P_1) \\ s_2 - s_1 &= c \ln(T_2/T_1) \end{aligned}$$

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Specific Heat of an Incompressible Substance

In a general sense, $u = u(T, v)$, which when differentiated will give:

$$du = \left.\frac{\partial u}{\partial T}\right|_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv \tag{1}$$

where the specific heat at constant volume is defined as:

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v \tag{2}$$

In a similar manner, $h = h(T, P)$ and

$$dh = \left.\frac{\partial h}{\partial T}\right|_P dT + \left.\frac{\partial h}{\partial P}\right|_T dP \tag{3}$$

where the specific heat at constant pressure is defined as:

$$c_P = \left(\frac{\partial h}{\partial T}\right)_P \tag{4}$$

Combining Eqs. 1-4

$$du = c_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv \tag{5}$$

$$dh = c_P dT + \left.\frac{\partial h}{\partial P}\right|_T dP \tag{6}$$

But we also know that $h = u + Pv$, which when differentiated gives

$$dh = du + d(Pv) = du + Pdv + vdP \tag{7}$$

but $dv \rightarrow 0$ for an incompressible substance.

Substituting Eq. 5 and 6 into Eq. 7, we obtain

$$c_P dT + \left.\frac{\partial h}{\partial P}\right|_T dP = c_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv + vdP \tag{8}$$

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But if we take the derivative of h with respect to P at constant temperature we obtain

$$\begin{aligned}\left.\frac{\partial h}{\partial P}\right|_T &= \left.\frac{\partial(u(T,v))}{\partial P}\right|_T + P\left.\frac{\partial v}{\partial P}\right|_T + v\left.\frac{\partial P}{\partial P}\right|_T \\ &= v \qquad (9)\end{aligned}$$

Therefore Eq. 8 can be written as

$$c_P dT + v dP = c_v dT + v dP \qquad (10)$$

Therefore

$$c_p = c_v = c \qquad (11)$$