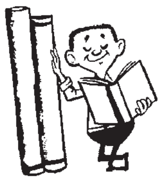
Availability



Reading	Problems
7.1 - 7.8	7-29, 7-37, 7-40, 7-48, 7-60, 7-68, 7-107

Second Law Analysis of Systems

AVAILABILITY:

- the theoretical maximum amount of work that can be obtained from a system at a given state $\boldsymbol{P_1}$ and $\boldsymbol{T_1}$ when interacting with a reference atmosphere at the constant pressure and temperature $\boldsymbol{P_0}$ and $\boldsymbol{T_0}$.
- describes the work potential of a given system.
- also referred to as "exergy".

The following observations can be made about availability:

1. Availability is a **property** - since any quantity that is fixed when the state is fixed is a property. For a system at state 1 and specified values of the atmosphere of $\boldsymbol{T_0}$ and $\boldsymbol{P_0}$, the maximum useful work that can be produced is fixed.
2. Availability is a **composite property** - since its value depends upon an external datum - the temperature and pressure of the dead state.
3. Availability of a system is **0** at its **dead state** when $\boldsymbol{T = T_0}$ and $\boldsymbol{P = P_0}$. It is not possible for the system to interact with the reference atmosphere at the dead state. The system is said to be in thermodynamic equilibrium with its surroundings.
4. Unless otherwise stated, assume the dead state to be:

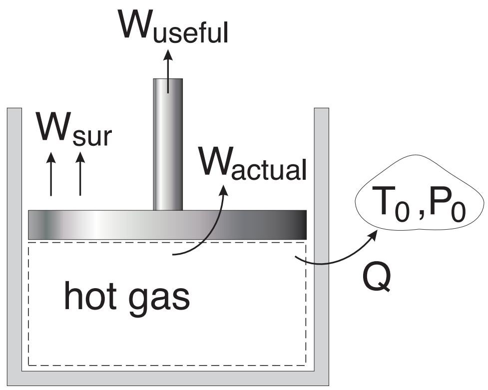
$$\boldsymbol{P_0 = 1\ atm}$$

$$\boldsymbol{T_0 = 25^\circ C}$$

5. The maximum work is obtained through a reversible process to the dead state.

$$\underbrace{\boldsymbol{REVERSIBLE\ WORK}}_{\boldsymbol{W_{rev}}} = \underbrace{\boldsymbol{USEFUL\ WORK}}_{\boldsymbol{W_{useful}}} + \underbrace{\boldsymbol{IRREVERSIBILITY}}_{\boldsymbol{I}}$$

Control Mass Analysis



$$W_{rev} = W_{useful} \; + \; I$$

where

$$\begin{aligned} W_{actual} &= W_{useful} + W_{sur} \\ W_{sur} &= P_0(V_2 - V_1) = -P_0(V_1 - V_2) \end{aligned}$$

To find W_{actual}, incorporate the 1st and 2nd laws to get

$$W_{actual} = (E_1 - E_2) - T_0(S_1 - S_2) - T_0\mathcal{P}_s$$

and

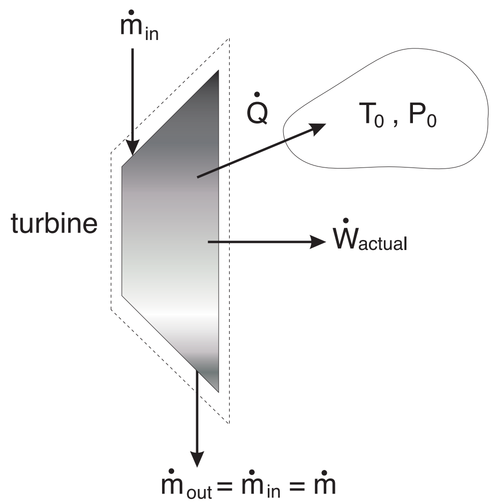
$$\begin{aligned} W_{useful} &= (E_1 - E_2) - T_0(S_1 - S_2) + P_0(V_1 - V_2) - T_0\mathcal{P}_s \\ W_{rev} &= (E_1 - E_2) - T_0(S_1 - S_2) + P_0(V_1 - V_2) \end{aligned}$$

$$\begin{aligned} X = \Phi &= CONTROL\ MASS\ AVAILABILITY \\ &= W_{rev}\ (in\ going\ to\ the\ dead\ state) \\ &= (E - E_0) - T_0(S - S_0) + P_0(V - V_0) \end{aligned}$$

The availability destroyed is $X_{des} = I = W_{rev} - W_{useful} = T_0\mathcal{P}_s$

This can be referred to as: irreversibilities, availability destruction or loss of availability.

Control Volume Analysis



From the 1st law

$$\cancelto{0}{\frac{dE_{cv}}{dt}} = -\dot{W}_{actual} - \dot{Q} + \left[\dot{m}(h + \frac{(v^*)^2}{2} + gz)\right]_{in} - \left[\dot{m}(h + \frac{(v^*)^2}{2} + gz)\right]_{out} \quad (1)$$

From the 2nd law

$$\cancelto{0}{\frac{dS_{cv}}{dt}} = \left(\dot{m}s + \frac{\cancelto{0}{\dot{Q}}}{T_{TER}}\right)_{in} - \left(\dot{m}s + \frac{\dot{Q}}{T_0}\right)_{out} + \dot{\mathcal{P}}_s \quad (2)$$

Combining (1) and (2) through the $\dot{Q}$ term,

$$\begin{aligned} \dot{W}_{actual} &= \left[\dot{m}\left(h + \frac{(v^*)^2}{2} + gz - T_0 s\right)\right]_{in} - \left[\dot{m}\left(h + \frac{(v^*)^2}{2} + gz - T_0 s\right)\right]_{out} - T_0\dot{\mathcal{P}}_S \\ &= \dot{m}\left[-T_0\Delta s + \Delta h + \Delta KE + \Delta PE\right] - (T_0\dot{\mathcal{P}}_s) \quad (3) \end{aligned}$$

The specific flow availability, ψ, is given as

$$\psi = -T_0(s - s_0) + (h - h_0) + \left(\frac{(v^*)^2}{2} - \frac{(\cancelto{0}{v_0^*})^2}{2}\right) + g(z - \cancelto{0}{z_0}) \quad (4)$$

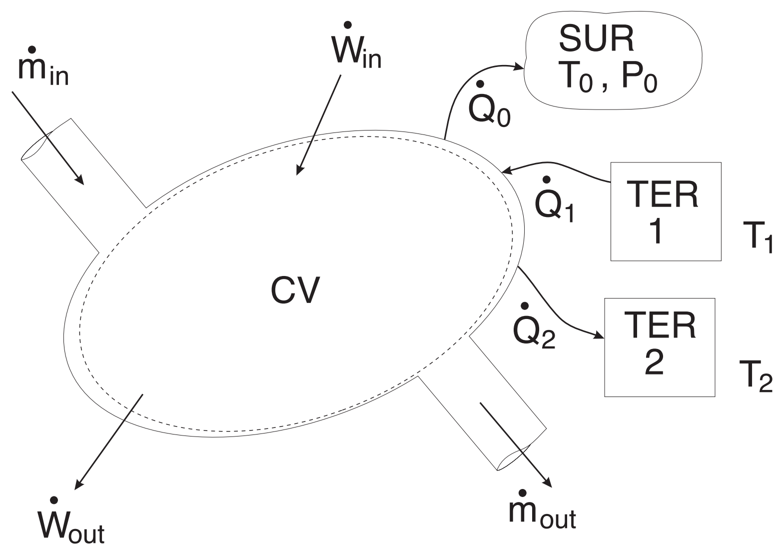
For a steady state, steady flow process where we assume KE=PE=0

$$\dot{W}_{rev} = (\dot{m}\psi)_{in} - (\dot{m}\psi)_{out} \quad (5)$$

$$\dot{X}_{des} = \dot{I} = \dot{W}_{rev} - \dot{W}_{actual} = T_0\dot{\mathcal{P}}_s \quad (6)$$

$$\psi = (h - h_0) - T_0(s - s_0) \quad (7)$$

The General Exergy Equation



From the 1st law

$$\frac{dE_{cv}}{dt} = \dot{W}_{in} - \dot{W}_{out} - \dot{Q}_0 + \dot{Q}_1 - \dot{Q}_2 + [\dot{m}(e + Pv)]_{in} - [\dot{m}(e + Pv)]_{out} \quad (1)$$

From the 2nd law

$$\frac{dS_{cv}}{dt} = \left(\dot{m}s - \frac{\dot{Q}_0}{T_0} + \frac{\dot{Q}_1}{T_1}\right)_{in} - \left(\dot{m}s + \frac{\dot{Q}_2}{T_2}\right)_{out} + \dot{\mathcal{P}}_s \quad (2)$$

Multiply (2) by $\boldsymbol{T_0}$ and subtract from (1) to eliminate $\boldsymbol{Q_0}$, which leads to the generalized exergy equation

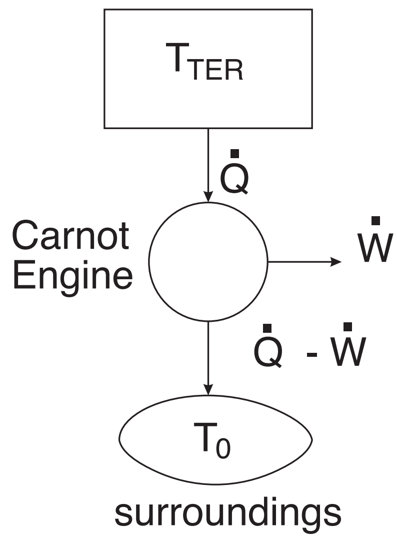
$$\begin{aligned} \frac{d}{dt}(E - T_0 S)_{CV} &= \dot{W}_{in} - \dot{W}_{out} + [\dot{m}(e + Pv - T_0 s)]_{in} \\ &\quad -[\dot{m}(e + Pv - T_0 s)]_{out} + \left(\dot{Q}_1 - \frac{T_0 \dot{Q}_1}{T_1}\right)_{in} \\ &\quad - \left(\dot{Q}_2 - \frac{T_0 \dot{Q}_2}{T_2}\right)_{out} - T_0 \dot{\mathcal{P}}_S \qquad (3) \end{aligned}$$

We can rewrite Eq. (3) in a generalized form by introducing the definitions of $\boldsymbol{\Phi}$ and $\boldsymbol{\psi}$.

$$\begin{aligned} \frac{d\Phi}{dt} &= P_0 \frac{dV_{CV}}{dt} + \left[\dot{W} + \dot{m}\psi + \dot{Q}\left(1 - \frac{T_0}{T_{TER}}\right)\right]_{in} \\ &\quad - \left[\dot{W} + \dot{m}\psi + \dot{Q}\left(1 - \frac{T_0}{T_{TER}}\right)\right]_{out} - \dot{I} \end{aligned}$$

where

$$\begin{aligned} \dot{I} = \dot{X}_{des} &= T_0 \dot{\mathcal{P}}_s \\ &= exergy\ destruction\ rate \\ \Phi &= [(E - E_0) + P_0(V - V_0) - T_0(S - S_0)]_{CV} \\ &= non - flow\ exergy \\ \psi &= (h - h_0) - T_0(s - s_0) + \frac{1}{2}\left[(v^*)^2 - (v_0^*)^2\right] + g(z - z_0) \\ &= flow\ exergy \\ \dot{W}_{useful} &= \underbrace{(\dot{W}_{in} - \dot{W}_{out})}_{\dot{W}_{actual}} - \left(\underbrace{P_0 \frac{dV_{CV}}{dt}}_{W_{sur}}\right) \end{aligned}$$



Notice that:

$$\boldsymbol{\eta = \frac{\dot{W}}{\dot{Q}} = 1 - \frac{T_0}{T_{TER}}}$$

Therefore

$$\boldsymbol{\dot{W} =} \qquad \underbrace{\boldsymbol{\dot{Q}\left(1 - \frac{T_0}{T_{TER}}\right)}}_{\substack{\text{appears in the} \\ \text{general exergy equation}}}$$

This term represents the work potential (exergy) of a given TER with respect to the surroundings (dead state) at $\boldsymbol{T_0}$.

Efficiency and Effectiveness

1. **First law efficiency (thermal efficiency)**

$$\eta = \frac{net\ work\ output}{gross\ heat\ input} = \frac{W_{net}}{Q_{in}}$$

Carnot cycle

$$\eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

2. **Second Law Efficiency (effectiveness)**

$$\eta_{2nd} = \frac{net\ work\ output}{maximum\ reversible\ work} = \frac{net\ work\ output}{availability}$$

Turbine $\rightarrow$ $\eta_{2nd} = \dfrac{\dot{W}/\dot{m}}{\psi_e - \psi_i}$

Compressor $\rightarrow$ $\eta_{2nd} = \dfrac{\psi_e - \psi_i}{\dot{W}/\dot{m}}$

Heat Source $\rightarrow$ $\eta_{2nd} = \dfrac{\dot{W}/\dot{m}}{\dfrac{\dot{Q}}{\dot{m}}\left[1 - \dfrac{T_0}{T_{TER}}\right]}$

3. **Isentropic efficiency (process efficiency)**

   (a) adiabatic turbine efficiency

$$\eta_T = \frac{work\ of\ actual\ adiabatic\ expansion}{work\ of\ reversible\ adiabatic\ expansion} = \frac{W_{act}}{W_S}$$

   (b) adiabatic compressor efficiency

$$\eta_C = \frac{work\ of\ reversible\ adiabatic\ compression}{work\ of\ actual\ adiabatic\ compression} = \frac{W_S}{W_{act}}$$