

Week 2: Lecture 2**Isentropic Processes for Ideal Gases**

Assume:

- constant specific heats over a wide range of temperature
- $ds = 0$
- $du = c_v dT \quad \equiv c_v = \left(\frac{\partial u}{\partial T} \right)_v$
- $dh = c_p dT \quad \equiv c_p = \left(\frac{\partial h}{\partial T} \right)_P$

For isentropic processes

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{k-1} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k}$$

where

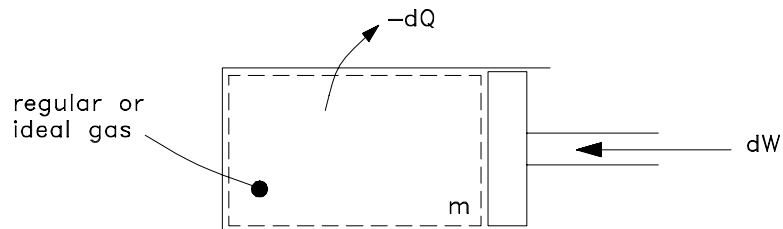
$$k = \frac{c_p}{c_v}$$

Polytropic Processes for Ideal Gases

The isentropic process is a special case of a more general process known as a polytropic process, where

$$Pv^n = \text{constant}$$

isothermal process	$n = 1$	$Pv = RT = \text{constant}$
isobaric process	$n = 0$	$Pv^0 = \text{constant}$
isentropic process	$n = k$	$k = c_p/c_v$
constant volume process	$n \rightarrow \infty$	$Pv^\infty = \text{constant}$

Week 2: Lecture 2**Reversible Compression/Expansion for a Fixed Mass:**

From the first law

$$mdu = dW - dQ \quad (1)$$

From the second law

$$mds = -\frac{dQ}{T} \quad (2)$$

Combining (1) and (2) through the dQ term

$$mdu = dW + mTds \quad (3)$$

From Gibb's equation

$$mdu = mTds - mPdv \quad (4)$$

Combining (3) and (4)

$$\frac{W_{1-2}}{m} = \int_1^2 -Pdv$$

This is suitable for reversible processes with any gas, real or ideal.

Week 2: Lecture 2**Summary**

For a perfect gas with constant specific heat

$$Pv = RT$$

$$u_2 - u_1 = c_v(T_2 - T_1)$$

$$h_2 - h_1 = c_p(T_2 - T_1)$$

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$= c_p \ln \frac{T_2}{T_1} + R \ln \frac{P_2}{P_1}$$

$$= c_p \ln \frac{v_2}{v_1} + R \ln \frac{P_2}{P_1}$$

$$R = c_p - c_v$$

For a reversible, isothermal process for an ideal gas

$$W_{1-2} = mRT \ln \frac{V_1}{V_2}$$

$$= mRT \ln \frac{P_2}{P_1}$$