Week 2: Lecture 1

Equations of State

Perfect Gas

The defining equation for a perfect gas is

$$\frac{Pv}{T} = \frac{PV}{Tm} \quad = \quad \text{constant} \quad = \quad R$$

where R is a constant for a particular gas, defined as

$$R = \mathcal{R}/\hat{M}$$

and $\mathcal{R}$ the universal gas constant is 8.314 $kJ/(kmole \cdot K)$, and $\hat{M}$ is the molal mass defined as:

$$\hat{M} = \frac{m}{n} = \frac{mass}{amount\ of\ substance\ (mole)} = (kg/kmole)$$

In practice, no gases obeys this law rigidly but most gases tend towards it. An imaginary ideal gas which obeys this law is called a perfect gas.

The perfect gas assumption is viable when

- temperatures are considerably in excess of the critical temperature of the fluid
- at very low pressures

Gibb's Equation

By combining a first law energy balance and the differential form of the entropy equation we can obtain Gibb's equation

$$dQ \quad = \quad dW + dU \quad (energy\ equation)$$

$$dS \quad = \quad dQ/T$$

$$\boxed{ds = \frac{du}{T} + \frac{Pdv}{T}}$$

Week 2: Lecture 2

Isentropic Processes for Ideal Gases

Assume:

- constant specific heats over a wide range of temperature
- $ds = 0$
- $du = c_v dT \quad \equiv c_v = \left(\frac{\partial u}{\partial T}\right)_v$
- $dh = c_p dT \quad \equiv c_p = \left(\frac{\partial h}{\partial T}\right)_P$

For isentropic processes

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

where

$$k = \frac{c_p}{c_v}$$

Polytropic Processes for Ideal Gases

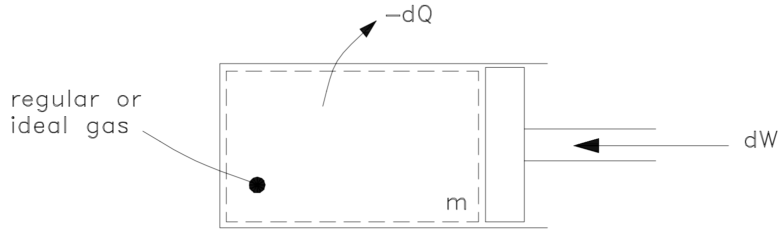
The isentropic process is a special case of a more general process known as a polytropic process, where

$$Pv^n = constant$$

isothermal process	$n = 1$	$Pv = RT = constant$
isobaric process	$n = 0$	$Pv^0 = constant$
isentropic process	$n = k$	$k = c_p/c_v$
constant volume process	$n \to \infty$	$Pv^\infty = constant$

Week 2: Lecture 2

Reversible Compression/Expansion for a Fixed Mass:



From the first law

$$mdu = dW - dQ \quad (1)$$

From the second law

$$mds = -\frac{dQ}{T} \quad (2)$$

Combining (1) and (2) through the dQ term

$$mdu = dW + mTds \quad (3)$$

From Gibb's equation

$$mdu = mTds - mPdv \quad (4)$$

Combining (3) and (4)

$$\frac{W_{1-2}}{m} = \int_1^2 -Pdv$$

This is suitable for reversible processes with any gas, real or ideal.

Week 2: Lecture 2

Summary

For a perfect gas with constant specific heat

$$
\begin{aligned}
Pv &= RT \\
u_2 - u_1 &= c_v(T_2 - T_1) \\
h_2 - h_1 &= c_p(T_2 - T_1) \\
s_2 - s_1 &= c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \\
&= c_p \ln \frac{T_2}{T_1} + R \ln \frac{P_2}{P_1} \\
&= c_p \ln \frac{v_2}{v_1} + R \ln \frac{P_2}{P_1} \\
R &= c_p - c_v
\end{aligned}
$$

For a reversible, isothermal process for an ideal gas

$$
\begin{aligned}
W_{1-2} &= mRT \ln \frac{V_1}{V_2} \\
&= mRT \ln \frac{P_1}{P_2}
\end{aligned}
$$

Week 2: Lecture 3

Incompressible Liquids

- a substance whose volume cannot be changed
- no substance is truly incompressible, but this model is good for most liquids and solids

A simple compressible substance has only one work mode and that work mode is Pdv work.

If a substance is assumed to be incompressible then its internal energy, for example, cannot be varied independently by work transfer but it can be varied by heat transfer at constant volume.

For a incompressible substance

$$c_p = c_v = c$$

From Gibb's equation we can see that

$$s_2 - s_1 = c \ln\left(\frac{T_2}{T_1}\right)$$

For an isothermal process ($T_2 = T_1$) we see that the process becomes isentropic ($S_2 - S_1$).

In summary

$$\begin{aligned}
u_2 - u_1 &= c(T_2 - T_1) \\
h_2 - h_1 &= (u_2 - u_1) + v(P_2 - P_1) \\
s_2 - s_1 &= c \ln(T_2/T_1)
\end{aligned}$$

Week 2: Lecture 3

Specific Heat of an Incompressible Substance

In a general sense, $u = u(T, v)$, which when differentiated will give:

$$du = \left.\frac{\partial u}{\partial T}\right|_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv \tag{1}$$

where the specific heat at constant volume is defined as:

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v \tag{2}$$

In a similar manner, $h = h(T, P)$ and

$$dh = \left.\frac{\partial h}{\partial T}\right|_P dT + \left.\frac{\partial h}{\partial P}\right|_T dP \tag{3}$$

where the specific heat at constant pressure is defined as:

$$c_P = \left(\frac{\partial h}{\partial T}\right)_P \tag{4}$$

Combining Eqs. 1-4

$$du = c_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv \tag{5}$$

$$dh = c_P dT + \left.\frac{\partial h}{\partial P}\right|_T dP \tag{6}$$

But we also know that $h = u + Pv$, which when differentiated gives

$$dh = du + d(Pv) = du + Pdv + vdP \tag{7}$$

but $dv \to 0$ for an incompressible substance.

Substituting Eq. 5 and 6 into Eq. 7, we obtain

$$c_P dT + \left.\frac{\partial h}{\partial P}\right|_T dP = c_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv + vdP \tag{8}$$

Week 2: Lecture 3

But if we take the derivative of h with respect to P at constant temperature we obtain

$$\begin{aligned} \left.\frac{\partial h}{\partial P}\right|_T &= \left.\frac{\partial (u(T,v))}{\partial P}\right|_T + P \left.\frac{\partial v}{\partial P}\right|_T + v \left.\frac{\partial P}{\partial P}\right|_T \\ &= v \end{aligned} \tag{9}$$

Therefore Eq. 8 can be written as

$$c_P dT + v dP = c_v dT + v dP \tag{10}$$

Therefore

$$c_p = c_v = c \tag{11}$$