ME 354 THERMODYNAMICS - 2

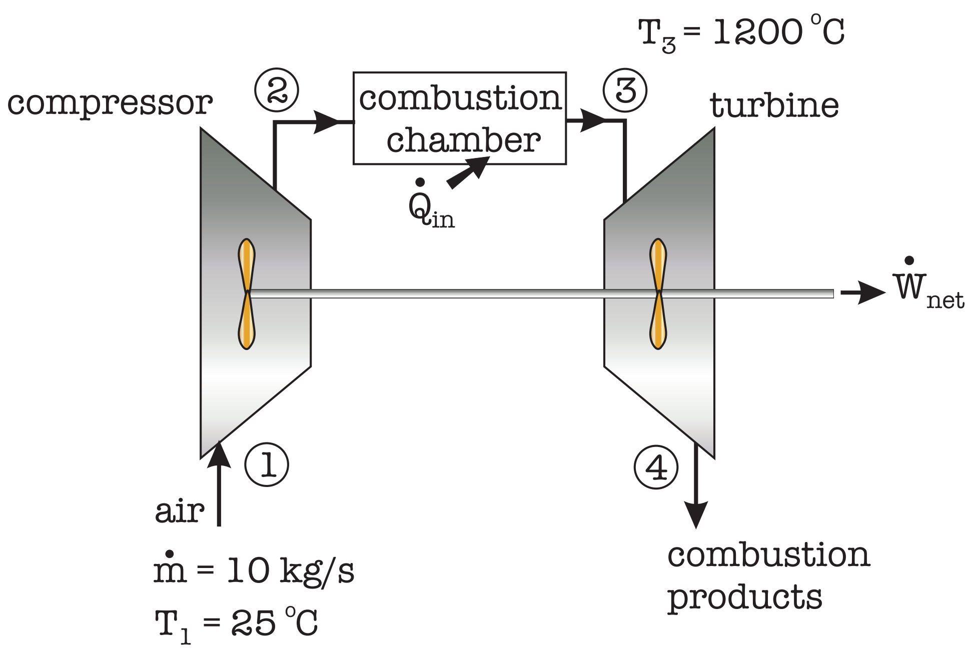
08 April 2004 Final Examination R. Culham

- This is a three-hour, closed-book examination.
- You are permitted to use one 8.5 in.× 11 in. crib sheet. (both sides) and the Property Tables and Figures from *Thermodynamics: An Engineering Approach*
- There are 5 questions to be answered. Read the questions very carefully.
- Clearly state all assumptions.
- It is your responsibility to write clearly and legibly.
- Good luck.

Question 1 (20 marks)

A gas turbine engine, as shown below, uses air as the working fluid with a constant specific heat of $c_p = 1.005\ kJ/(kg \cdot K)$ and $k = 1.4$. The compressor has an isentropic efficiency of **85%** and a pressure ratio of **18 : 1**, while the turbine has an isentropic efficiency of **90%** and a pressure ratio of **18 : 1**. Find the following:

i) the net power output (kW)
ii) the overall thermal efficiency of the engine
iii) the thermal efficiency of an equivalent ideal cycle
iv) the optimum pressure ratio to maximize work output for an ideal cycle



Using the isentropic relationships, $\boldsymbol{T_{2s}}$ can be calculated as

$$T_{2s} = T_1 \times (r_p)^{k-1/k} = (25 + 273) \times (18)^{0.4/1.4} = 680.56\ K$$

Since the compressor efficiency is 85%, we can find $\boldsymbol{T_2}$ as follows

$$\begin{aligned} \eta_c &= \frac{T_{2s} - T_1}{T_2 - T_1} \\ 0.85 &= \frac{680.56 - 298}{T_2 - 298} \end{aligned}$$

$$T_2 = 748.1\ K$$

Using the isentropic relationship for the turbine

$$T_{4s} = T_3 \times \left(\frac{1}{r_p}\right)^{k-1/k} = (1200 + 273) \times \left(\frac{1}{18}\right)^{0.4/1.4} = 645.0\ K$$

Since the mass flow rate and the specific heat are constant, the isentropic efficiency of the turbine can be written as

$$\begin{aligned} \eta_T &= \frac{T_3 - T_4}{T_3 - T_{4s}} \\ 0.90 &= \frac{1473 - T_4}{1473 - 645.0} \end{aligned}$$

Therefore

$$T_4 = 1473 - 0.90(1473 - 645.0) = 727.8\ K$$

The net work output per unit mass is

$$\begin{aligned} \frac{W_{net}}{m} &= c_p(T_3 - T_4) - c_p(T_2 - T_1) \\ &= 1.005\ kJ/kg \cdot K\ [(1473 - 727.8) - (748.1 - 298)]\ K \\ &= 296.58\ kJ/kg \end{aligned}$$

Part i)

The power output is

$$\boldsymbol{Power = \dot{m} \times \frac{W_{net}}{m} = 10\ kg/s\ \times\ 296.58\ kJ/kg = 2965.8\ kW \Leftarrow}$$

Part ii)

The thermal efficiency of the engine is

$$\begin{aligned}\eta &= \frac{W_{net}}{Q_{in}} = \frac{c_p(T_3 - T_4) - c_p(T_2 - T_1)}{c_p(T_3 - T_2)} \\ &= \frac{(1473 - 727.8) - (748.1 - 298)}{(1473 - 748.1)} \\ &= 0.407 = 40.7\% \Leftarrow\end{aligned}$$

Note: $\eta = 1 - (r_p)^{1-k/k}$ can only be used for the ideal Brayton cycle where $\eta_c = \eta_T = 1$

Part iii)

The thermal efficiency for an ideal Brayton cycle is given by

$$\begin{aligned}\eta &= 1 - (r_p)^{(1-k)/k} \\ &= 1 - (18)^{(1-1.4)/1.4} \\ &= 0.5621 \Leftarrow\end{aligned}$$

Part iv)

The optimum pressure ratio is

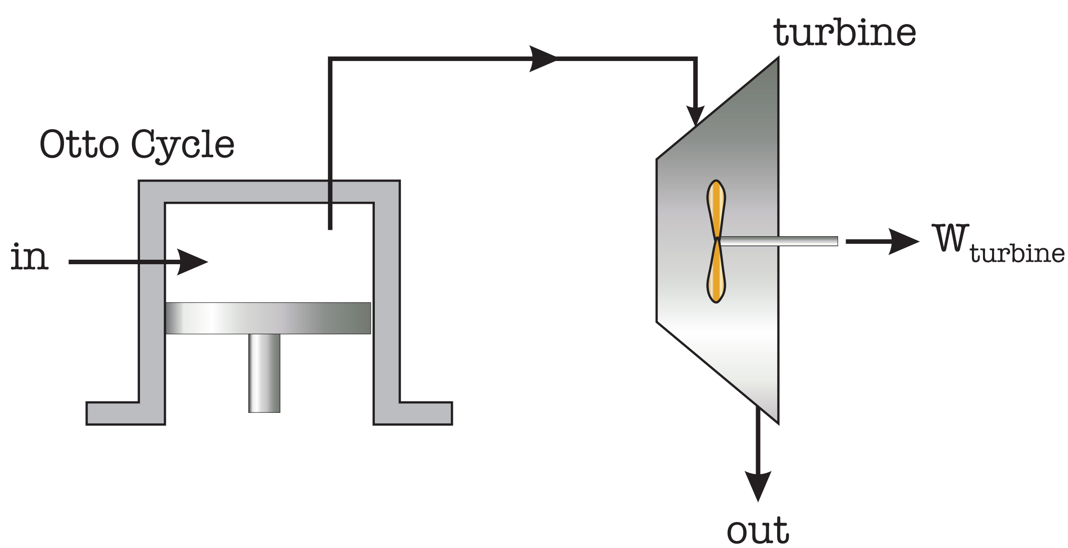
$$r_{p,opt} = \left(\frac{T_3}{T_1}\right)^{k/(2k-2)} = \left(\frac{1473}{298}\right)^{1.4/.8} = 16.39 \Leftarrow$$

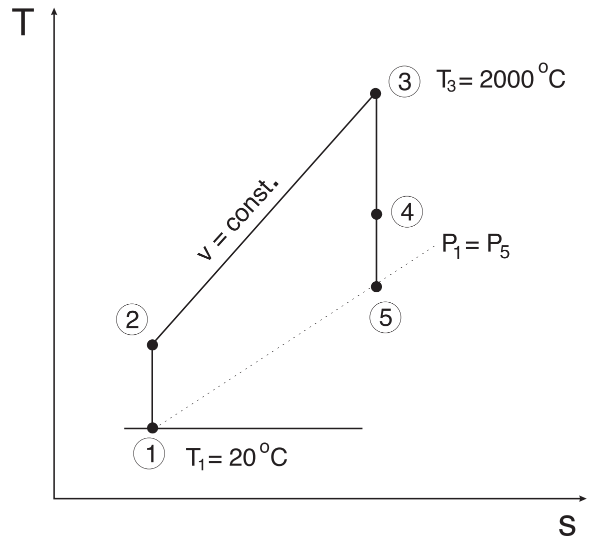
Question 2 (20 marks)

An open, ideal Otto-cycle engine has a compression ratio of **10 : 1**. The air just prior to the compression stroke is at $\mathbf{20°}\boldsymbol{C}$ and $\mathbf{100}\ \boldsymbol{kPa}$. The maximum cycle temperature is $\mathbf{2000\ °}\boldsymbol{C}$. The thermal efficiency of the ideal, Otto cycle is **0.60**.

Rather than simply discharging the air to the atmosphere after expansion in the cylinder, an isentropic turbine is installed in the exhaust to produce additional work. Assume constant specific heats, the mass flow rate through the turbine is steady and the pressure at the inlet to the cylinder is identical to the pressure at the discharge of the turbine.

i) draw a $\boldsymbol{T - s}$ process diagram for the compound engine
ii) determine the work output of the turbine, ($\boldsymbol{kJ/kg}$)
iii) determine the overall thermal efficiency of the compound engine





Part i)

For an isentropic process

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{(k-1)} \quad \text{and} \quad \frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{(k-1)} = \left(\frac{P_4}{P_3}\right)^{(k-1)/k}$$

Therefore

$$T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{(k-1)} = 293\ K\ (10)^{1.4-1} = 735.98\ K$$

and

$$T_4 = T_3 \left(\frac{v_3}{v_4}\right)^{(k-1)} = T_3 \left(\frac{v_4}{v_3}\right)^{-(k-1)}$$

But we know for an IC engine,

$$\frac{v_4}{v_3} = \frac{v_1}{v_2}$$

Therefore

$$T_4 = T_3 \left(\frac{v_4}{v_3}\right)^{-(k-1)} = T_3 \left(\frac{v_1}{v_2}\right)^{-(k-1)}$$

and

$$T_4 = (2000 + 273)\ K \quad \times (10)^{-(1.4-1)} = 904.9\ K$$

The net work output from the Otto cycle is

$$\begin{aligned} w_{net,otto} &= c_p(T_3 - T_4) - c_p(T_2 - T_1) \\ &= 1.005\ kJ/(kg \cdot K)(2273 - 904.9 - 735.98 + 293)\ K \\ &= 929.75\ kJ/kg \end{aligned}$$

We have and isentropic process between 3 and 5, and we can write

$$\frac{T_5}{T_3} = \left(\frac{P_5}{P_3}\right)^{(k-1)/k}$$

Since the air behaves as and ideal gas, and we know that $P_1 = P_5 = P_{atm}$, we can write

$$\begin{aligned} P_3 &= R \cdot T_3/v_3 \\ P_1 = P_5 &= R \cdot T_1/v_1 \end{aligned}$$

Therefore

$$\frac{T_5}{T_3} = \left(\frac{P_5}{P_3}\right)^{(k-1)/k} = \left(\frac{R \cdot T_1}{v_1} \cdot \frac{v_3}{R \cdot T_3}\right)^{(k-1)/k}$$

But for an Otto cycle we have constant volume heat addition between 2 and 3 and $v_2 = v_3$

$$\frac{T_5}{T_3} = \left(\frac{R \cdot T_1}{v_1} \cdot \frac{v_2}{R \cdot T_3}\right)^{(k-1)/k} = \left(\frac{T_1}{T_3} \cdot \frac{v_2}{v_1}\right)^{(k-1)/k}$$

Therefore

$$T_5 = (2000 + 273)\ K \quad \left(\frac{(20 + 273)\ K}{(2000 + 273)\ K} \cdot \frac{1}{10}\right)^{0.4/1.4} = 655.7\ K$$

The work output of the turbine is

$$\begin{aligned} w_{turbine} &= (h4 - h5) = c_p(T_4 - T_5) \\ &= 1.005\ kJ/(kg \cdot K)(904.9 - 655.7)\ K \\ &= 250.4\ kJ/kg \Leftarrow \end{aligned}$$

Part ii)

The thermal efficiency of the compound engine is given by

$$\eta_{th} = \frac{w_{net,otto} + w_{turbine}}{q_h}$$

where

$$q_h = (w_{net,otto})/\eta_{otto} = 929.75\ (kJ/kg)/0.6 = 1549.58\ kJ/kg$$

Therefore

$$\eta_{th} = (929.75\ kJ/kg + 250.4\ kJ/kg)/1549.58\ kJ/kg = 0.762 \Leftarrow$$

Question 3 (20 marks)

A centrifugal compressor is installed in a natural gas pipeline to overcome the line friction pressure drop. The gas, which is **25%** hydrogen and **75%** methane by volume, enters the compressor at **20 °C** and **100 kPa** and leaves at **200 kPa**. Assuming a reversible, adiabatic process and that the properties are independent of temperature;

i) determine the outlet mixture temperature, (°C)
ii) determine the work required to drive the compressor, (kJ/kg)
iii) determine the final partial pressures, (kPa)
iv) determine the change in entropy of the hydrogen and the methane. Verify that the overall process is isentropic.

Part i)

Since the volume fractions are given for the gases, we can also derive the mole fractions

$$\frac{V_i}{V} = \frac{n_i}{n}$$

$$X_{H_2} = \frac{n_{H_2}}{n} = 0.25 \qquad\qquad X_{CH_4} = \frac{n_{CH_4}}{n} = 0.75$$

The mass fractions can be determined using

$$Y_i = X_i \left[\frac{\tilde{M}_i}{\sum_{i=1}^{2} X_i \tilde{M}_i} \right]$$

$$Y_{H_2} = 0.25 \left[\frac{2.016}{(0.25 \times 2.016) + (0.75 \times 16.043)} \right] = .0402$$

$$Y_{CH_4} = 0.75 \left[\frac{16.043}{(0.25 \times 2.016) + (0.75 \times 16.043)} \right] = .9598$$

Since the compression process is isentropic we know

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = \left(\frac{P_2}{P_1} \right)^{R/c_p}$$

But since we do not know k for the mixture, we must calculate

$$(k-1)/k = \frac{R}{c_p}$$

where

$$\begin{aligned} c_p &= Y_{H_2}(c_p)_{H_2} + Y_{CH_4}(c_p)_{CH_4} \\ &= .0402 \times 14.307 + .9598 \times 2.2537 = 2.738\ kJ/(kg \cdot K) \\ R &= Y_{H_2}R_{H_2} + Y_{CH_4}R_{CH_4} \\ &= .0402 \times 4.1240 + .9598 \times 0.5182 = 0.663\ kJ/(kg \cdot K) \end{aligned}$$

Therefore

$$T2 = (20 + 273.2)\ K\ \times\ \left(\frac{200}{100}\right)^{0.663/2.738} = 346.8\ K = 73.6\ ^{\circ}C$$

Part ii)

We can perform an energy balance on the compressor to find the work requirement of the compressor.

$$h_1 + \dot{w}_c = h_2$$

$$\dot{w}_c = h_2 - h_1 = c_p(T_2 - T_1) = 2.738\ \frac{kJ}{(kg \cdot K)}(346.8 - 293.2)\ K = 146.76\ \frac{kJ}{kg}$$

Part iii)

$$\begin{aligned} P_{H_2} &= X_{H_2}\ P_2 = 0.25 \times 200\ kPa = 50\ kPa \\ P_{CH_4} &= X_{CH_4}\ P_2 = 0.75 \times 200\ kPa = 150\ kPa \end{aligned}$$

Part iv)

The increase in entropy for the hydrogen is given as

$$\begin{aligned} s_2 - s_1 &= c_{p_{H_2}}\ \times\ \ln\left(\frac{T_2}{T_1}\right) - R\ \times\ \ln\left(\frac{P_2}{P_1}\right) \\ &= 14.307 \times \ln\left(\frac{346.8}{293.2}\right) - 4.124 \times \ln\left(\frac{50}{25}\right) \\ &= -0.4565\ \frac{kJ}{kg \cdot K} \end{aligned}$$

$$
\begin{aligned}
s_2 - s_1 &= c_{p_{CH_4}} \times \ln\left(\frac{T_2}{T_1}\right) - R \times \ln\left(\frac{P_2}{P_1}\right) \\
&= 2.2537 \times \ln\left(\frac{346.8}{293.2}\right) - 0.5182 \times \ln\left(\frac{150}{75}\right) \\
&= 0.0192 \frac{kJ}{kg \cdot K}
\end{aligned}
$$

To verify the compressor is isentropic

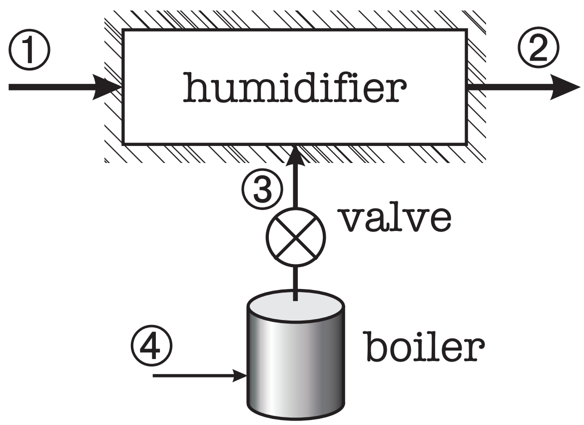
$$
\begin{aligned}
\Delta s &= Y_{H_2} \Delta s_{H_2} + Y_{CH_4} \Delta s_{CH_4} \\
&= 0.0402 \times (-0.4565) \frac{kJ}{(kg \cdot K)} + 0.9598 \times (0.0192) \frac{kJ}{(kg \cdot K)} \\
&= 0.00 \frac{kJ}{(kg \cdot K)}
\end{aligned}
$$

If Δs of the mixture is zero, therefore the compressor must be isentropic.

Question 4 (20 marks)

Moist air enters an adiabatic humidifier at state 1 at $\mathbf{20\ ^\circ C}$ and **10%** relative humidity with a volumetric flow rate of $\mathbf{0.25\ m^3/s}$. The air leaves the humidifier at state 2 at $\mathbf{22.5\ ^\circ C}$ and **70%** relative humidity. The change in the condition of the moist air is brought about by the injection of steam at state 3 from a boiler. The boiler is supplied with $\mathbf{22.5\ ^\circ C}$ water at state 4. An adiabatic valve between the boiler and the humidifier causes the steam pressure to drop from boiler pressure to the humidifier pressure of $\mathbf{101.325\ kPa}$.

i) determine the mass flow rate [$\boldsymbol{kg/hr}$] of the water at state point 4
ii) determine the heat transfer rate [$\boldsymbol{kW}$] to the boiler
iii) determine the pressure [$\boldsymbol{kPa}$] in the boiler when the temperature of the superheated steam is $\mathbf{300\ ^\circ C}$



We can use either the psychrometric chart or the controlling psychrometric equations to determine the properties at each state.

Part i)
First we note that since the mass of air at state 1 is equivalent to the mass of air at state 2

$$\dot{m}_{a,1} = \dot{m}_{a,2} = \dot{m}_a$$

and from Table A-4, $P_{sat,1}(20\ ^\circ C) = 2.339\ kPa$ and $P_{sat,2}(22.5\ ^\circ C) = 2.754\ kPa$.

Performing a mass balance for the water over the entire system we get

$$\dot{m}_a\omega_1 + \dot{m}_{w,4} = \dot{m}_a\omega_2 \qquad \Rightarrow \qquad \dot{m}_{w,4} = \dot{m}_a(\omega_2 - \omega_1)$$

$$\omega = 0.622\left(\frac{\phi P_{sat}}{P - \phi P_{sat}}\right)$$

$$\omega_1 = 0.622\left(\frac{0.10 \times 2.339\ kPa}{101.325\ kPa - 0.10 \times 2.339\ kPa}\right) = 0.00144\ \frac{kg_{H_2O}}{kg_{air}}$$

$$\omega_2 = 0.622\left(\frac{0.70 \times 2.754\ kPa}{101.325\ kPa - 0.70 \times 2.754\ kPa}\right) = 0.01206\ \frac{kg_{H_2O}}{kg_{air}}$$

The volumetric flow rate must be converted to mass flow rate as

$$v_a = \frac{R_a T_1}{P_{a,1}} = \frac{R_1 T_1}{(P - \phi_1 P_{sat,1})} = \frac{(0.287)(293)}{(101.325 - 0.1 \times 2.339)} = 0.831\ \frac{m^3}{kg}$$

and

$$\dot{m}_a = \frac{\dot{V}_1}{v_a} = \frac{0.25\ m^3/s}{0.831\ m^3/kg} = 0.30\ kg/s$$

$$\begin{aligned} \dot{m}_{w,4} = \dot{m}_a(\omega_2 - \omega_1) &= (0.30\ kg_{air}/s)(0.01206 - 0.00144)\ \frac{kg_{H_2O}}{kg_{air}} \\ &= 0.003186\ \frac{kg_{H_2O}}{s} = 11.47\ \frac{kg_{H_2O}}{hr} \Leftarrow \end{aligned}$$

Part ii)

Performing an energy balance over the system gives us

$$\dot{m}h_1^* + \dot{m}_{w,4}h_{w,4} + \dot{q} = \dot{m}_a h_2^*$$

Solving for $\dot{q}$

$$\begin{aligned} \dot{q} &= \dot{m}_a(h_2^* - h_1^*) - \dot{m}_{w,4}h_{w,4} \\ &= 0.30\ kg/s(53.7 - 24.0)\ kJ/kg - 0.003186\ kg/s(94.425\ kJ/kg) \\ &= 8.61\ kW \Leftarrow \end{aligned}$$

Part iii)

Performing an energy balance over just the humidifier

$$\dot{m}h_1^* + \dot{m}_{w,3}h_{w,3} = \dot{m}_a h_2^*$$

or by noting that $\dot{m}_{w,3} = \dot{m}_{w,4}$

$$\begin{aligned} h_{w,3} &= \frac{\dot{m}_a(h_2^* - h_1^*)}{\dot{m}_{w,4}} \\ &= \frac{0.30\ kg/s(53.7 - 24.0)\ kJ/kg}{0.003186\ kg/s} = 2797\ kJ/kg \end{aligned}$$

From Table A-6, we can find the pressure of superheated steam when $T = 300\ °C$ and the enthalpy is $2796.6\ kJ/kg$

$$P_3 \approx 800\ kPa \Leftarrow$$

Question 5 (20 marks)

An abandoned gas storage tank contains a mixture of ethane (5% by volume) and air (Hint: air consists of $\mathbf{1}$ ***kmole*** of $\boldsymbol{O_2}$ and $\mathbf{3.76}$ ***kmoles*** of $\boldsymbol{N_2}$ for each ***kmole*** of fuel). The mixture is at $\mathbf{25\ °}\boldsymbol{C}$ and $\mathbf{1}$ ***atm***.

An eight-year old boy, standing on the tank lighting his first cigarette, excitedly throws the match into the storage tank. Assuming complete combustion happens so fast that the process occurs adiabatically at constant volume;

i) determine the final temperature [$\boldsymbol{K}$]
ii) determine the pressure, [$\boldsymbol{atm}$], inside the tank
iii) determine the air/fuel ratio on a per mass basis

Part i)

Air:	$\mathbf{4 \times 4.76}\ \boldsymbol{kmol} = \mathbf{19.04}\ \boldsymbol{kmol}$
Fuel:	$\mathbf{1}\ \boldsymbol{kmol}$
Total:	$\mathbf{20.04}\ \boldsymbol{kmol}$

$$\%\boldsymbol{fuel} = \mathbf{1/20.04 = 5\%}$$

The balanced reaction equation is

$$\boldsymbol{C_2H_6 + 4[O_2 + 3.76N_2] \rightarrow 2CO_2 + 3H_2O + 0.5O_2 + 15.04N_2}$$

An energy balance on the system gives

$$\boldsymbol{\sum n_P(\overline{h}_f^o + \overline{h} - \overline{h}^o)_P = \sum n_R(\overline{h}_f^o + \overline{h} - \overline{h}^o)_R}$$

Since the reactants are at $\mathbf{25\ °}\boldsymbol{C}$ and $\mathbf{1}$ ***atm***, the enthalpy of formation for oxygen and nitrogen are zero.

From the tables

Substance	$\mathbf{\overline{h}_f^o}$ **kJ/kmol**	$\mathbf{\overline{h}_{298\ K}}$ **kJ/kmol**
$\boldsymbol{C_2H_6}$ ***(g)***	-84,680	—
$\boldsymbol{O_2}$	0	8,682
$\boldsymbol{N_2}$	0	8,669
$\boldsymbol{H_2O}$ ***(g)***	-241,820	9,904
$\boldsymbol{CO_2}$	-393,520	9,364

Thus the enthalpy of the products must equal the enthalpy of the reactants

$$\begin{aligned}(2)(-393,520 + \overline{h}_{CO_2} - 9364) + (3)(-241,820 + \overline{h}_{H_2O} - 9904)& \\ +(0.5)(0 + \overline{h}_{O_2} - 8682) + (15.04)(0 + \overline{h}_{N_2} - 8669) &= (1)(-84,680)\end{aligned}$$

It yields

$$2\overline{h}_{CO_2} + 3\overline{h}_{H_2O} + 0.5\overline{h}_{O_2} + 15.04\overline{h}_{N_2} = 1,610,982.76\ kJ$$

A first guess of the adiabatic flame temperature is obtained by dividing the right-hand side of the equation by the total number of moles of products, which yields $\mathbf{1,610,982.76/(20.54) \approx 78,431\ kJ/kmol}$.

Since the majority of the products consists of N_2, look in the N_2 for an approximate temperature.

At $\mathbf{2350\ K}$

$$\begin{aligned}2\overline{h}_{CO_2} + 3\overline{h}_{H_2O} + 0.5\overline{h}_{O_2} + 15.04\overline{h}_{N_2} &= (2)(122,091) + (3)(100,846) \\ &+(0.5)(81,243) + (15.04)(77,496) = 1,752,881\ kJ\end{aligned}$$

Value is high, so try a lower temperature at $\mathbf{2150\ K}$

$$\begin{aligned}2\overline{h}_{CO_2} + 3\overline{h}_{H_2O} + 0.5\overline{h}_{O_2} + 15.04\overline{h}_{N_2} &= (2)(109,898) + (3)(90,330) \\ &+(0.5)(73,573) + (15.04)(70,226) = 1,583,772\ kJ\end{aligned}$$

By interpolation

$$T_p = T_{AD} = 2,182.2\ K \Leftarrow$$

Part ii)
The final pressure is

$$P_2 = \frac{n_2 \cdot \mathcal{R} \cdot T_2}{V_2}$$

but since $V_2 = V_1$, and $V_1 = n_1 \cdot \mathcal{R} \cdot T_1/P_1$, we can write

$$P_2 = \frac{n_2 \cdot \mathcal{R} \cdot T_2}{n_1 \cdot \mathcal{R} \cdot T_1/P_1} = \frac{T_2}{T_1} P_1 = \frac{2182.2\ K}{298}(1\ atm) = 7.32\ atm \Leftarrow$$

Part iii)

$$AF = \frac{m_{air}}{m_{fuel}} = \frac{(n\tilde{M})_{air}}{(n\tilde{M})_{C_2H_6}} = \frac{(4 \times 4.76)(28.97)}{(1)(30.070)} = 18.34\ \frac{kg_{air}}{kg_{CH_4}} \Leftarrow$$