ME 354 THERMODYNAMICS - 2

9 February 2004

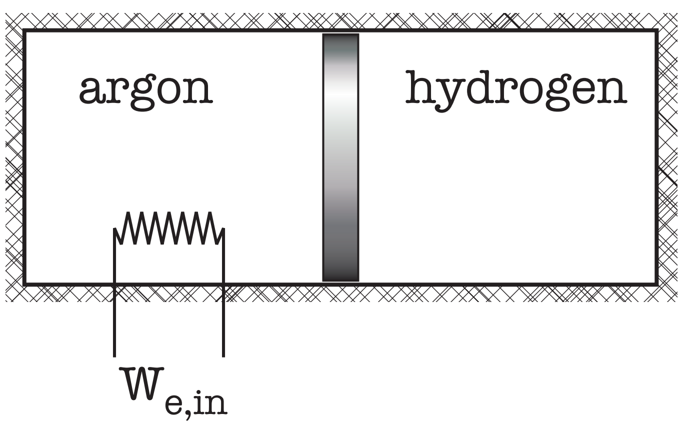
Midterm Examination   R. Culham

- This is a two-hour, closed-book examination.
- You are permitted to use one 8.5 in.× 11 in. crib sheet. (one side only) and the Property Tables and Figures from *Thermodynamics: An Engineering Approach*
- There are 3 questions to be answered. Read the questions very carefully.
- Clearly state all assumptions.
- It is your responsibility to write clearly and legibly.

Question 1 (20 marks)

Two well insulated chambers initially have equal volumes of $\boldsymbol{1\ m^3}$ and contain argon and hydrogen, respectively. The chambers are separated by a frictionless, adiabatic piston. Both gases are initially at $\boldsymbol{20\ °C}$ and $\boldsymbol{150\ kPa}$. An electrical resistance heater transfers energy to the argon until the pressure of both gases reaches $\boldsymbol{300\ kPa}$. The hydrogen can be assumed to undergo a reversible process. Assume constant specific heats (@ $\boldsymbol{300\ K}$) and assume the dead state conditions to be $\boldsymbol{T_0 = 20\ °C}$ and $\boldsymbol{P_0 = 100\ kPa}$.

a) Determine the final temperature of the argon in ($\boldsymbol{K}$).
b) Determine the electrical input in ($\boldsymbol{kJ}$).
c) Determine the availability destruction ($\boldsymbol{kJ}$) in the system.



$$
\begin{aligned}
V_{1A} &= 1.0\ m^3 & \quad V_{1H} &= 1.0\ m^3\\
T_{1A} &= 293\ K & T_{1H} &= 293\ K\\
P_{1A} &= 150\ kPa & P_{1H} &= 150\ kPa\\
P_{2A} &= 300\ kPa & P_{2H} &= 300\ kPa
\end{aligned}
$$

from Table A-2

$$
R_A = 0.2081\ kJ/kg\cdot K \qquad R_H = 4.1240\ kJ/kg\cdot K
$$

Using the ideal gas equation, the mass of the argon and the hydrogen can be found as follows:

$$
m_A = \left(\frac{PV}{RT}\right)_A = \frac{150\ kPa\ \cdot\ \dfrac{1\ kJ/m^3}{1\ kPa}\ \cdot\ 1\ m^3}{0.2081\ \dfrac{kJ}{kg\cdot K}\ \cdot\ 293\ K} = 2.460\ kg
$$

$$
m_H = \left(\frac{PV}{RT}\right)_H = \frac{150\ kPa\ \cdot\ \dfrac{1\ kJ/m^3}{1\ kPa}\ \cdot\ 1\ m^3}{4.124\ \dfrac{kJ}{kg\cdot K}\ \cdot\ 293\ K} = 0.124\ kg
$$

Part a)

Since the hydrogen is reversible and adiabatic, it is assumed to be isentropic. Therefore

$$
\frac{T_{2H}}{T_{1H}} = \left(\frac{P_{2H}}{P_{1H}}\right)^{(k-1)/k}
$$

From Table A-2, $k = 1.405$ for hydrogen

$$
T_{2H} = 293\ K\left(\frac{300}{150}\right)^{.405/1.405} = 357.80\ K
$$

From the ideal gas equation

$$
V_{2H} = \frac{m_H R_H T_{2H}}{P_{2H}} = \frac{0.124\ kg\ \cdot\ 4.124\ \dfrac{kJ}{kg\cdot K}\ \cdot\ 357.8\ K}{300\ kPa\ \cdot\ \dfrac{kJ/m^3}{kPa}} = 0.6099\ m^3
$$

The volume of argon is then

$$
V_{2A} = V_{total} - V_{2H} = 2\ m^3 - 0.6099\ m^3 = 1.3901\ m^3
$$

and the temperature of argon is

$$
T_{2A} = \frac{P_{2A} V_{2A}}{m_A R_A} = \frac{300\ kPa\ \cdot\ \dfrac{kJ/m^3}{kPa}\ \cdot\ 1.3901\ m^3}{2.460\ kg\ \cdot\ 0.2081\ \dfrac{kJ}{kg\cdot K}} = 814.63\ K \Leftarrow \underline{part\ a}
$$

Part b)

The change in internal energy in the argon is

$$\Delta U_A = m_A \cdot c_{v_A} \cdot (T_{2A} - T_{1A}) = 2.460\, kg \cdot 0.3122\, \frac{kJ}{kg \cdot K} \cdot (814.63\, K - 293\, K) = 400.62\, kJ$$

The change in internal energy in the hydrogen is

$$\Delta U_H = m_H \cdot c_{v_H} \cdot (T_{2H} - T_{1H}) = 0.124\, kg \cdot 10.183\, \frac{kJ}{kg \cdot K} \cdot (357.8\, K - 293\, K) = 81.82\, kJ$$

Perform an energy balance over the system, where the electrical input is $W_{e,in}$.

$$\Delta W_{e,in} = \Delta U_A + \Delta U_H = 400.62\ kJ + 81.82\ kJ = 482.44\ kJ \Leftarrow \underline{part\ b}$$

Part c)

The production of entropy in the system is given by

$$\mathcal{P}_S = \Delta S_A + \Delta S_H$$

But since the hydrogen is isentropic, $\Delta S_H = 0$
Therefore

$$\mathcal{P}_S = \Delta S_A = m_A(s_{2A} - s_{1A})$$

The change is entropy between states $1 \longrightarrow 2$ is given by

$$\begin{aligned} s_{2A} - s_{1A} &= c_{P_A} \ln\left(\frac{T_{2A}}{T_{1A}}\right) - R_A \ln\left(\frac{P_{2A}}{P_{1A}}\right) \\ &= 0.5203\ \frac{kJ}{kg \cdot K} \ln\left(\frac{814.63\ K}{293\ K}\right) - 0.2081 \frac{kJ}{kg \cdot K} \ln\left(\frac{300\ kPa}{150\ kPa}\right) \\ &= 0.388\ \frac{kJ}{kg \cdot K} \end{aligned}$$

The availability destruction is

$$\begin{aligned} X_{destruction} &= T_0 \mathcal{P}_S = T_0 \ \cdot\ m_A \ \cdot\ (s_{2A} - s_{1A}) \\ &= 293\ K \ \cdot\ 2.460\ kg \ \cdot\ 0.388\ \frac{kJ}{kg \cdot K} = 279.66\ kJ \Leftarrow \underline{part\ c} \end{aligned}$$

Question 2 (20 marks)

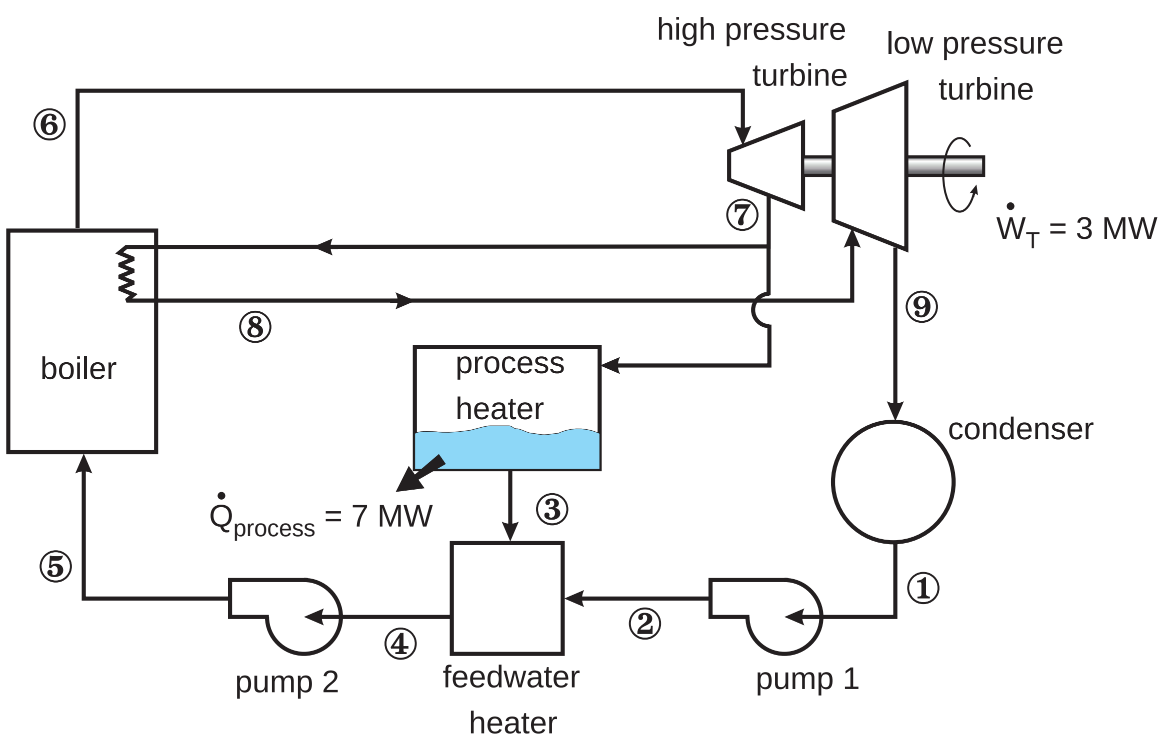
Consider a cogeneration power plant which is modified with reheat and which produces **3 MW** of power and supplies **7 MW** of process heat. Steam enters the high-pressure turbine at **8 MPa** and **500 °C** and expands to a pressure of **1 MPa.** At this pressure, part of the steam is extracted from the turbine and routed to the process heater, while the remainder is reheated to **500 °C** and expanded in the low-pressure turbine to the condenser pressure of **15 kPa**. The condensate from the condenser is pumped to **1 MPa** and is mixed with the extracted steam, which leaves the process heater as a subcooled liquid at **120 °C**. The mixture is then pumped to the boiler pressure.

Assume:

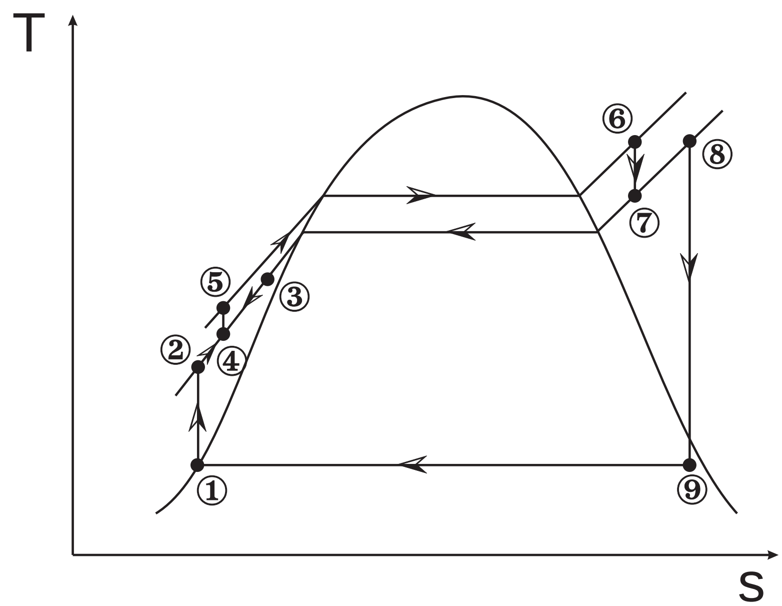
1. both turbines are isentropic
2. perfect mixing in the feedwater heater
3. constant specific heats
4. no internal irreversibilies
5. neglect pump work in your calculations

Find:

a) draw the complete process on a $T - s$ diagram and clearly label all state points
b) determine the rate of heat input in the boiler, [MW]
c) determine the mass fraction of steam extracted for process heating.



Part a:



Part b:

State	P $[MPa]$	T $[°C]$	h $[kJ/kg]$	s $[kJ/kg \cdot K]$	Comments
1	0.015	53.97	225.94		saturated liquid
2	1.0		225.94		neglect pump work
3	1.0	120	504.56		compressed liquid
4	1.0		441.38		
5	8.0		441.38		$h_5 = h_4$, neglect pump work
6	8.0	500	3398.3	6.7240	
7	1.0	206.5	2842.8	6.7240	$s_7 = s_6$, isentropic
8	1.0	500	3478.5	7.7622	
9	1.0		2518.41	7.7622	$s_9 = s_8$, isentropic

State Point 1: exit of the condenser

Assume a saturated liquid at the exit of the condenser where $P_1 = P_9 = 15\ kPa$.

$$\begin{aligned} T_{sat@15\ kPa} &= 53.97\ °C \\ v_1 &= 0.001014\ m^3/kg \\ h_1 &= 225.94\ kJ/kg \end{aligned}$$

Since we have assumed that the work added by the pump is negligible we can write

$$h_2 = h_1 = 225.94\ kJ/kg$$

State Point 6: inlet to the high pressure turbine, using Table A-6

$$\begin{aligned} P_6 &= 8.0\ MPa \\ T_6 &= 500\ ^\circ C \\ h_6 &= 3398.3\ kJ/kg \\ s_6 &= 6.7240\ kJ/kg \cdot K \end{aligned}$$

Since the turbine is isentropic, $s_6 = s_7 = 6.7240\ kJ/kg \cdot K$. Interpolate within Table A-6 to find h_7.

$$\begin{aligned} T_7 &= 206.5\ ^\circ C \\ h_7 &= (0.86996)(2827.9\ kJ/kg) + (0.13004)(2942.6\ kJ/kg) = 2842.8\ kJ/kg \end{aligned}$$

State Point 8: inlet to the low pressure turbine

$$\begin{aligned} T_8 &= 500\ ^\circ C \\ P_8 &= 1\ MPa \\ h_8 &= 3478.5\ kJ/kg \\ s_8 &= 7.7622\ kJ/kg \cdot K \end{aligned}$$

State Point 9: outlet of the low pressure turbine

Since the turbine is isentropic

$$s_9 = s_8 = 7.7622\ kJ/kg \cdot K$$

This is clearly within the vapor dome and we can determine the quality at state point 9 as

$$x_9 = \frac{7.7622 - 0.7549}{8.0085 - 0.7549} = 0.966$$

Therefore the enthalpy at 9 is

$$\begin{aligned} h_9 &= (1 - x_9)h_f + x_9 h_g \\ &= 0.034(225.94\ kJ/kg) + 0.966(2599.1\ kJ/kg) = 2518.41\ kJ/kg \end{aligned}$$

State Point 3: exit of the process heater

The exit of the process heater sees a saturated, compressed liquid at $T_3 = 120\ ^\circ C$. We can determine the enthalpy of a compressed liquid by integrating the following:

$$dh = \overline{c}dT + vdP$$

From the saturated liquid tables

$$\begin{aligned} P_{sat@120\ ^\circ C} &= 0.19853\ MPa \\ h_{f@120\ ^\circ C} &= 503.71\ kJ/kg \\ v_{f@120\ ^\circ C} &= 0.001060\ m^3/kg \end{aligned}$$

Therefore

$$\begin{aligned} h_3 &= h_{f@120\ ^\circ C} + v_{f@120\ ^\circ C}(P_3 - P_{sat@120\ ^\circ C}) \\ &= 503.71\ \frac{kJ}{kg} + 0.001060\ \frac{m^3}{kg}(1000 - 198.53)\ kPa \\ &= 504.56\ kJ/kg \end{aligned}$$

We can perform a mass balance over the processor heater to find $\dot{m}_3$. Noting that the enthalpy into the process heater is $\dot{m}_3 \cdot h_7$ since the mass flow rate into, is the same as the mass flow rate out of the process heater (steady flow assumption).

$$\dot{m}_3 h_7 = \dot{Q}_{process} + \dot{m}_3 h_3$$

$$\begin{aligned} \dot{m}_3 &= \frac{\dot{Q}_{process}}{(h_7 - h_3)} \\ &= \frac{7000\ kW \cdot (1\ kJ/s/kW)}{(2842.8 - 504.56)\ kJ/kg} \\ &= 2.994\ kg/s \end{aligned}$$

We can now perform an energy balance over the two turbines to find the mass flow rate at state point 8.

$$\dot{m}_6(h_6 - h_7) + \dot{m}_8(h_8 - h_9) = \dot{W}_T$$

but

$$\dot{m}_6 = \dot{m}_7 + \dot{m}_3 \qquad \text{and} \qquad \dot{m}_8 = \dot{m}_7$$

Therefore we can write

$$\begin{aligned} \dot{m}_7 &= \frac{\dot{W}_T - \dot{m}_3(h_6 - h_7)}{(h_6 - h_7) + (h_8 - h_9)} \\ &= \frac{3000\ kW \cdot (1\ kJ/s/kW) - (2.994\ kg/s)(3398.3 - 2842.8)\ kJ/kg}{(3398.3 - 2842.8)\ kJ/kg + (3478.5 - 2518.41)\ kJ/kg} \\ &= 0.882\ kg/s \end{aligned}$$

and

$$\dot{m}_6 = \dot{m}_7 + \dot{m}_3 = 0.88\ kg/s\ +\ 2.994\ kg/s = 3.874\ kg/s$$

State Point 4: exit of the FWH

Performing an energy balance we can write

$$\dot{m}_3 h_3 + \dot{m}_2 h_2 = \dot{m}_4 h_4$$

but

$$\dot{m}_4 = \dot{m}_6 \qquad \text{and} \qquad \dot{m}_2 = \dot{m}_7$$

Therefore

$$\begin{aligned} h_4 &= \frac{\dot{m}_3 h_3 + \dot{m}_7 h_2}{\dot{m}_6} \\ &= \frac{(2.994\ kg/s)(504.56\ kJ/kg) + (0.882\ kg/s)(225.94\ kJ/kg)}{3.874\ kg/s} \\ &= 441.38\ kJ/kg \end{aligned}$$

If we assume the work of the pump 2 is negligible, then $h_5 = h_4$.
Performing and energy balance over the boiler we get

$$\begin{aligned} \dot{Q}_{boiler} &= \dot{m}_6(h_6 - h_5) + \dot{m}_7(h_8 - h_7) \\ &= 3.87\ kg/s(3398.3 - 441.38)kJ/kg + 0.882\ kg/s(3478.5 - 2842.8)\ kJ/s \\ &= 12,004\ kJ/s = 12\ mW \Leftarrow \underline{part\ b} \end{aligned}$$

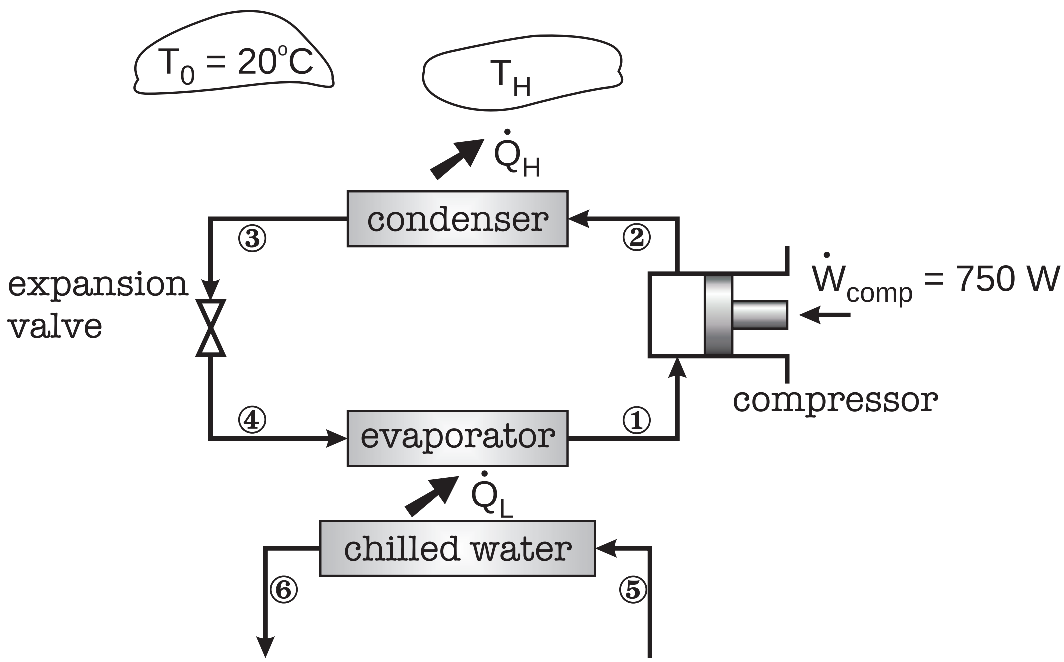
The fraction of the steam extracted for the process heating is

$$\frac{\dot{m}_3}{\dot{m}_6} = \frac{2.994\ kg/s}{3.87\ kg/s} = 0.774 = 77.4\% \Leftarrow \underline{part\ c}$$

Question 3 (20 marks)

A vapor compression refrigerator using R-134a is to provide chilled water at **5 °*C***. Originally the water is at **20 °*C***. The pressure in the evaporator is **280 *kPa*** and in the condenser **700 *kPa***. The R-134a enters the compressor at **0 °*C*** and leaves at **50 °*C***. The work transfer rate to the adiabatic, compressor is **750 *W***. The R-134a leaves the condenser as a saturated liquid. Assume the dead state to be **100 *kPa*** and **20 °*C***.

a) Determine the COP of the refrigerator.
b) Determine the mass flow rate [***kg/hr***] of the chilled water that can be produced at the prescribed temperature.
c) Determine the rate of availability destruction [***W***] in the compressor and the expansion valve.
d) Determine the second law efficiency of the cycle. Explain why in some instances the second law efficiency in a refrigeration cycle can appear to be negative.



State Point	T (°C)	P (kPa)	x	h (kJ/kg)	s (kJ/kgK)
1	0	280		247.64	0.9238
2	50	700		286.35	0.9867
3	26.72	700		86.78	0.3242
4		280	0.1938	86.78	0.3323
5	20				
6	5				

From Table A-12 in Cengel and Boles, state point 3 given as a saturated liquid has a pressure of $\boldsymbol{P_3 = 700\ kPa}$ and a corresponding saturation temperature of $\boldsymbol{T_3 = 26.72\ °C}$. This allows us to get the enthalpy at state point 3 which is identical to the enthalpy on the other side of the expansion valve at state point 4. Therefore

$$\begin{aligned} \boldsymbol{h_4} &= \boldsymbol{(1-x)h_f + xh_g} \\ \boldsymbol{86.78} &= \boldsymbol{(1-x) \cdot 48.39 + x \cdot 246.52} \end{aligned}$$

Solving gives

$$\boldsymbol{x = 0.1938}$$

The entropy at state point 4 can be calculated as

$$\begin{aligned} \boldsymbol{s_4} &= \boldsymbol{(1-x)s_f + xs_g} \\ &= \boldsymbol{(1-0.1938) \cdot 0.1911 + 0.1938 \cdot 0.9197} \\ &= \boldsymbol{0.3323\ kJ/kgK} \end{aligned}$$

Part a)

The coefficient of performance is given as

$$\boldsymbol{COP = \frac{benefit}{cost} = \frac{\dot{Q}_L/\dot{m}}{\dot{W}_{comp}/\dot{m}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{247.64 - 86.78}{286.35 - 247.64} = 4.16 \Leftarrow \underline{part\ a}}$$

Part b)

Performing an energy balance over the evaporator gives

$$\boldsymbol{\dot{m} \cdot (h_1 - h_4) = \dot{m}_{cw} c_p (T_5 - T_6)}$$

or

$$\boldsymbol{\dot{m}_{cw} = \dot{m} \frac{h_1 - h_4}{c_p(T_5 - T_6)}}$$

The mass flow rate of the refrigerant, $\boldsymbol{\dot{m}}$ can be determined as

$$\boldsymbol{\dot{m} = \frac{\dot{W}_{comp}}{h_2 - h_1} = \frac{0.75\ kJ/s}{(286.35 - 247.64) kJ/kg} = 0.019325\ kg/s = 69.75\ kg/hr}$$

Then using the c_p of water at $25\ ^\circ C$

$$\dot{m}_{cw} = 69.75\ kg/hr \left(\frac{247.64 - 86.78}{4.18\ kJ/kg \times (20-5)\ K} \right) = 178.94\ kg/hr \Leftarrow \underline{part\ b}$$

Part c)
The availability destruction in the compressor is given as

$$\begin{aligned} \dot{X}_{c_{destroyed}} = T_0 \dot{\mathcal{P}}_s &= T_0 \dot{m}(s_2 - s_1) \\ &= 293\ K \ \cdot\ 0.019325\ kg/s\ \cdot (0.9867 - 0.9238)\ kJ/kg \cdot K \\ &= 0.356\ kW = 356\ W \end{aligned}$$

The availability destruction in the expansion valve is given as

$$\begin{aligned} \dot{X}_{v_{destroyed}} = T_0 \dot{\mathcal{P}}_s &= T_0 \dot{m}(s_4 - s_3) \\ &= 293\ K \ \cdot\ 0.019325\ kg/s\ \cdot (0.3323 - 0.3242)\ kJ/kg \cdot K \\ &= 0.0459\ kW = 45.9\ W \Leftarrow \underline{part\ c} \end{aligned}$$

Part d)
The second law efficiency is given as

$$\eta_{2nd} = \frac{benfit}{cost} = \frac{|\psi_1 - \psi_4|}{w_{comp}}$$

The change in availability across the evaporator is

$$\psi_1 - \psi_4 = (h_1 - h_4) - T_0(s_1 - s_4) = (247.64 - 86.78) - 293(0.9238 - 0.3323) = -12.45\ kJ/kg$$

Therefore

$$\eta_{2nd} = \frac{|\psi_1 - \psi_4|}{\dot{W}_{comp}/\dot{m}} = \frac{12.45}{38.71} = 0.322 \Leftarrow \underline{part\ d}$$

The change in availability across the evaporator is proportional to

$$\Delta\psi \propto q_L \left[1 - \frac{T_0}{T_L} \right]$$

Since $T_L < T_0$, the change in $\Delta\psi$ will be negative and in turn the second law efficiency will appear to be negative.