University of Waterloo

ME 354

THERMODYNAMICS 2
MIDTERM EXAMINATION

February 14, 2011 5:30 pm - 7:30 pm

Instructor: R. Culham

Name: ______________________________

Student ID Number: ______________________________

Instructions

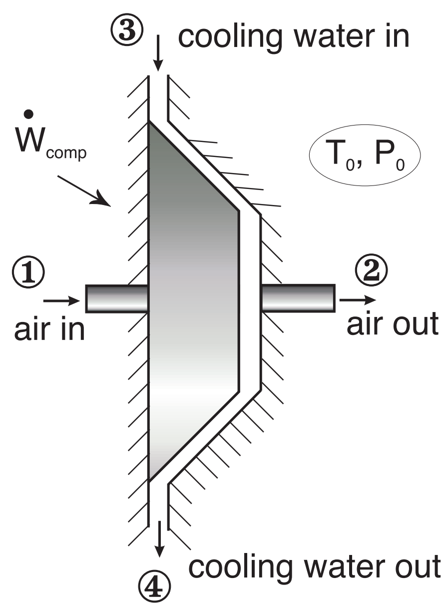
1. This is a 2 hour, closed-book examination.
2. Answer all questions in the space provided. If additional space is required, use the back of the pages or the blank pages included.
3. Permitted aids include:
   - Property Tables Booklet (Fundamentals of Thermodynamics, Borgnakke and Sonntag, 7th ed) or a photocopy of this booklet
   - one 8.5 in.× 11 in. crib sheet. (one side only)
   - calculator
4. It is your responsibility to write clearly and legibly. Clearly state all assumptions. Part marks will be given for part answers, provided that your methodology is clear.

Question	**Marks**	**Grade**
1	17	
2	20	
3	18	
TOTAL	55	

Question 1 (17 marks)

A compressor fitted with a water jacket and operating at steady state conditions takes in air with a volumetric flow rate of $\mathbf{900}\ \boldsymbol{m^3/h}$ at $\mathbf{300}\ \boldsymbol{K}$ and $\mathbf{95}\ \boldsymbol{kPa}$ and discharges air at $\mathbf{590}\ \boldsymbol{K}$ and $\mathbf{800}\ \boldsymbol{kPa}$. Cooling water enters the water jacket at $\mathbf{20}\ \boldsymbol{°C}$, $\mathbf{100}\ \boldsymbol{kPa}$ with a mass flow rate of $\mathbf{1400}\ \boldsymbol{kg/h}$ and exits at $\mathbf{30}\ \boldsymbol{°C}$ and essentially the same pressure. There is no significant heat transfer from the outer surface of the water jacket to its surroundings and kinetic and potential energy effects can be ignored. Properties can be assumed to be constant with respect to temperature. The dead state is assumed to be $\boldsymbol{T_0 = 25\ °C}$ and $\boldsymbol{P_0 = 1\ atm}$.

a) Determine the rate of exergy input [$\boldsymbol{kW}$] into the system.
b) Determine the change in flow exergy rate [$\boldsymbol{kW}$] for the air stream.
c) Determine the change in flow exergy rate [$\boldsymbol{kW}$] for the water stream.
d) Determine the rate of exergy destruction [$\boldsymbol{kW}$] in the process.



Assumptions

1. steady state
2. $KE = PE \approx 0$
3. the boundary between the water jacket and the surroundings is adiabatic
4. air is modeled as an ideal gas
5. water is modeled as an incompressible liquid
6. $T_0 = 298\ K, \quad P_0 = 1\ atm$

Part a)
From conservation of mass

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_{air} \qquad \text{and} \qquad \dot{m}_3 = \dot{m}_4 = \dot{m}_{water}$$

From conservation of energy

$$\dot{W}_{comp} = \dot{m}_{air}(h_2 - h_1) + \dot{m}_{water}(h_4 - h_3)$$

The specific volume of the air at the inlet can be determined as

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\ kJ/kg \cdot K)(300\ K)}{(95\ kPa)\left(\dfrac{1\ kJ/m^3}{kPa}\right)} = 0.906\ m^3/kg$$

The mass flow rate is

$$\dot{m}_{air} = \frac{\mathcal{V}_1}{v_1} = \frac{900\ m^3/h}{0.906\ m^3/kg} = 993.03\ kg/h = 0.276\ kg/s$$

From Table A7.1 at $T_1 = 300\ K$ and $T_2 = 590\ K$

$$h_1 = 300.47\ kJ/kg$$

$$h_2 = 596.84\ kJ/kg$$

For compressed water we know (we have assumed $P_4 \approx P_3$)

$$(h4 - h3) = \bar{c}(T_4 - T_3) + v(P_4 - P_3) = (4.18\ kJ/kg \cdot K)(30 - 20)\ K = 41.8\ kJ/kg$$

<u>or</u> from Table B.1.4 at $P = 500\ kPa$

$$h_4 - h_3 = 41.8\ kJ/kg$$

Therefore

$$\dot{W}_{comp} = \left(0.276\ \frac{kg}{s}\right)(596.84 - 300.47)\ \frac{kJ}{kg} + \left(1400\ \frac{kg}{h}\right)\left(\frac{1\ h}{3600\ s}\right)\left(41.8\ \frac{kJ}{kg}\right)$$

$$= 98.05\ kW \Leftarrow$$

Since electrical work input is entropy free, the work of the compressor is the maximum, reversible work input and therefore the rate of exergy input into the system.

Part b)
The change in flow exergy rate in the air, from inlet to exit is

$$\dot{m}_{air}(\psi_2 - \psi_1) \;=\; \dot{m}_{air}\left[(h_2 - h_1) - T_0(s_2 - s_1)\right]$$

where $s^o(T)$ can be obtained from Table A7.1

$$\begin{aligned} s_2 - s_1 &= s^o(T_2) - s^o(T_1) - R\ln(P_2/P_1) \\ &= \left(7.55861 - 6.86926 - 0.287\ln\left(\frac{800}{95}\right)\right)\; kJ/kg\cdot K \\ &= 0.07783\; kJ/kg\cdot K \end{aligned}$$

Therefore

$$\begin{aligned} \dot{m}_{air}(\psi_2 - \psi_1) &= \dot{m}_{air}[(h_2 - h_1) - T_0(s_2 - s_1)] \\ &= \left(0.276\;\frac{kg}{s}\right)\left[(596.84 - 300.47)\;\frac{kJ}{kg} - 300\;K \times (0.07783\; kJ/kg\cdot K)\right] \\ &= 76.35\; kW \Leftarrow \end{aligned}$$

Part c)
The change in flow exergy rate in the water, from inlet to exit is

$$\dot{m}_{water}(\psi_4 - \psi_3) \;=\; \dot{m}_{water}\left[(h_4 - h_3) - T_0(s_4 - s_3)\right]$$

where

$$\begin{aligned} s_4 - s_3 &= \bar{c}\ln\left(\frac{T_4}{T_3}\right) \\ &= \left(4.18\;\frac{kJ}{kg\cdot K}\right)\ln\left(\frac{30+273}{20+273}\right) \\ &= 0.14\; kJ/kg\cdot K \end{aligned}$$

Therefore

$$\begin{aligned} \dot{m}_{water}(\psi_4 - \psi_3) &= \left(1400\;\frac{kg}{h}\right)\left(\frac{1\;h}{3600\;s}\right)\left[41.8\;\frac{kJ}{kg} - 298\;K \times (0.14\; kJ/kg\cdot K)\right] \\ &= 0.031\; kW \Leftarrow \end{aligned}$$

Part d)
The rate of exergy destruction is given as

$$\dot{X}_{des} = T_0 \; \dot{S}_{gen}$$

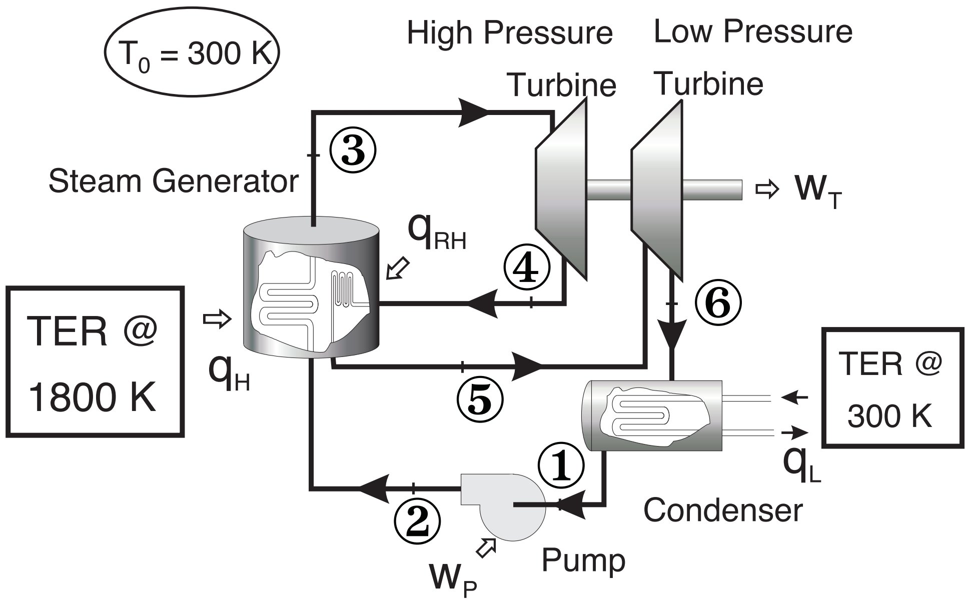
or from a simple exergy balance with the information we have already determined.

$$\begin{aligned}
\textbf{Exergy in} &= \textbf{Exergy out} \\
\dot{W}_{comp} &= \dot{m}_{air}(\psi_2 - \psi_1) + \dot{m}_{water}(\psi_4 - \psi_3) + \dot{X}_{des} \\
\dot{X}_{des} &= 98.05 - 76.35 - 0.031 = 21.67 \; kW \Leftarrow
\end{aligned}$$

Question 2 (20 marks)

A steam power plant operates as an ideal reheat Rankine cycle. Steam enters the high pressure turbine at **8 MPa** and **500 °C** and leaves at **3 MPa**. Steam is then reheated at a constant pressure to **500 °C** before it expands to **20 kPa** in the low pressure turbine. Assume that heat is being added to the boiler from a high temperature source at **1800 K** and the condenser is transferring heat to a temperature sink at **300 K**. Determine:

a) the total work output of the turbines, [kJ/kg]
b) the thermal efficiency of the cycle
c) the entropy generation at each of the heat transfer devices, [$kJ/(kg \cdot K)$]



Assumptions

1. steady flow, steady state
2. $KE = PE = 0$
3. ideal Rankine cycle, internally irreversible

State	T (°C)	P (kPa)	v (m^3/kg)	h (kJ/kg)	s ($kJ/kg \cdot K$)	Comments
1		20	0.001017	251.38	0.8319	saturated water
2		8000		259.50	0.8320	
3	500	8000		3398.27	6.7239	Table B.1.3
4		3000		3104.0	6.7239	$s_4 = s_3$
5	500	3000		3456.48	7.2337	
6		20		2384.7	7.2337	$s_6 = s_5$

Part a)

State point 1 is assumed to be a saturated liquid in an ideal Rankine cycle, therefore from Table A-5

$$h_1 = h_{f@20\ kPa} = 251.38\ kJ/kg$$

and

$$v_1 = v_{f@20\ kPa} = 0.001017\ m^3/kg$$

and

$$s_1 = s_2 = s_{f@20\ kPa} = 0.8319\ kJ/kg \cdot K$$

The work of the pump is given as

$$w_P = v_1(P_2 - P_1) = (0.001017\, m^3/kg)(8000 - 20)kPa \times \left(\frac{1\ kJ}{1\ kPa \cdot m^3}\right) = 8.12\, kJ/kg$$

$$h_2 = h_1 + w_p = 251.38 + 8.12 = 259.50\ kJ/kg$$

At the entry to the high pressure turbine, from Table B.1.3

$$\left.\begin{array}{l} P_3 = 8\ MPa \\ T_3 = 500\ °C \end{array}\right\} \begin{array}{l} h_3 = 3398.27\ kJ/kg \\ s_3 = 6.7239\ kJ/kg \cdot K \end{array}$$

Assuming an isentropic high pressure turbine, we note that $s_4 = s_3$ and that $s_4 > s_{f@3\ MPa}$, therefore state point 4 is in the superheated region.

$$\left.\begin{array}{l} P_4 = 3\ MPa \\ s_4 = s3 = 6.7239\ kJ/kg \cdot K \end{array}\right\} h_4 = 3104.0\ kJ/kg$$

At the entry to the low pressure turbine, from Table A-6

$$\left.\begin{array}{l} P_5 = 3\ MPa \\ T_5 = 500\ °C \end{array}\right\} \begin{array}{l} h_5 = 3456.48\ kJ/kg \\ s_5 = 7.2337\ kJ/kg \cdot K \end{array}$$

Assuming an isentropic low pressure turbine, we note that $s_5 = s_6$ and that $s_6 < s_{f@20\ kPa}$, therefore state point 6 is under the dome.

$$P_6 = 20\ kPa \qquad s_6 = s_5 = 7.2337\ \frac{kJ}{kg \cdot K} \quad \left\} \begin{array}{l} x_6 = \dfrac{s_6 - s_f}{s_g - s_f} = \dfrac{7.2337 - 0.8319}{7.9085 - 0.8319} = 0.9046 \\ h_6 = h_f + x_6(h_g - h_f) \\ \quad = 251.38 + (0.9046)(2609.7 - 251.38) = 2384.7\ kJ/kg \end{array} \right.$$

The total turbine work output is

$$w_T = (h_3 - h_4) + (h_5 - h_6) = (3398.27 - 3104.0) + (3456.48 - 2384.7) = 1366.1\ kJ/kg \Leftarrow$$

Part b)

The heat input to the cycle is given as

$$q_H = (h_3 - h_2) = 3398.27 - 259.50 = 3138.77\ kJ/kg$$

$$q_{RH} = (h_5 - h_4) = 3456.48 - 3104.0 = 352.48\ kJ/kg$$

$$q_{in,total} = q_H + q_{RH} = 3491.25\ kJ/kg$$

The net work output is given as

$$w_{net} = w_T - w_P = 1366.1 - 8.12 = 1357.98\ kJ/kg$$

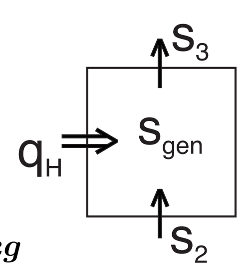
and the cycle efficiency is given as

$$\eta = \frac{w_{net}}{q_{in}} = \frac{1357.98\ kJ/kg}{3491.25\ kJ/kg} = 38.9\% \Leftarrow$$

Part c)

Performing an entropy balance between state point 2 and state point 3 we get

$$s_2 + \frac{q_H}{T_H} + s_{gen} = s_3$$

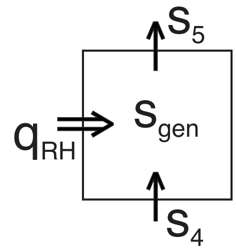


and

$$\begin{aligned} s_{gen} &= s_3 - s_2 - \frac{q_H}{T_H} \\ &= 6.7239\ \frac{kJ}{kg \cdot K} - 0.8319\ \frac{kJ}{kg \cdot K} - \frac{3138.77\ kJ/kg}{1800\ K} \\ &= 4.148\ \frac{kJ}{kg \cdot K} \Leftarrow \end{aligned}$$

Similarly on the reheat coil, we can perform an entropy balance between state point 4 and state point 5

$$s_4 + \frac{q_{RH}}{T_H} + s_{gen} = s_5$$



and

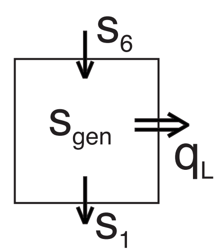
$$\begin{aligned} s_{gen} &= s_5 - s_4 - \frac{q_{RH}}{T_H} \\ &= 7.2337 \ \frac{kJ}{kg \cdot K} - 6.7239 \ \frac{kJ}{kg \cdot K} - \frac{352.48 \ kJ/kg}{1800 \ K} \\ &= 0.3140 \ \frac{kJ}{kg \cdot K} \Leftarrow \end{aligned}$$

The heat loss at the condenser is given as

$$q_L = (h_6 - h_1) = 2384.7 - 251.38 = 2133.32 \ kJ/kg$$

An entropy balance over the compressor between state point 6 and sate point 1 gives

$$s_6 + s_{gen} = s_6 + \frac{q_L}{T_L}$$



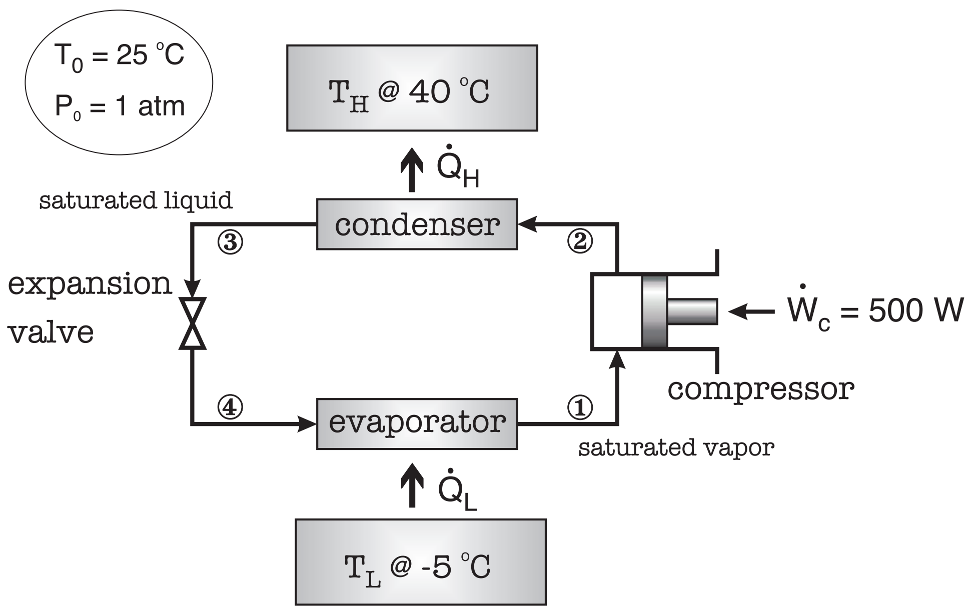
and

$$\begin{aligned} s_{gen} &= s_1 - s_6 + \frac{q_{6-1}}{T_L} \\ &= 0.8319 \ \frac{kJ}{kg \cdot K} - 7.2337 \ \frac{kJ}{kg \cdot K} + \frac{2133.32 \ kJ/kg}{300 \ K} \\ &= 0.7093 \ \frac{kJ}{kg \cdot K} \Leftarrow \end{aligned}$$

Question 3 (18 marks)

A vapor compression refrigerator using R-134a as the working fluid requires **500 *W*** of electrical power to drive the isentropic compressor. Both the evaporator and the condenser are assumed to be 100% effective, that is to say the low temperature TER and the saturated vapor leaving the evaporator are both at $\boldsymbol{-5\ ^{\circ}C}$ and the high temperature TER and the saturated liquid leaving the condenser are at $\boldsymbol{40\ ^{\circ}C}$. The dead state is assumed to be $\boldsymbol{T_0 = 25\ ^{\circ}C}$ and $\boldsymbol{P_0 = 1\ atm}$. Determine:

a) the rate of heat transfer in the evaporator and the condenser, $[\boldsymbol{kJ/kg}]$
b) the COP of the cycle
c) the change in availability in the cold temperature space and the high temperature space $[\boldsymbol{kJ/kg}]$
d) the second law efficiency for the refrigerator
e) the total cycle exergy destruction , $[\boldsymbol{kW}]$



Assumptions

1. isentropic compressor
2. $\boldsymbol{KE = PE = 0}$
3. internally irreversible
4. throttling valve is adiabatic

State Point 1: The exit of the evaporator is assumed to be at a saturated vapor. From Table B.5 at $\boldsymbol{T = -5\ °C}$

$$\boldsymbol{h_1 = 395.34\ kJ/kg} \quad \text{and} \quad \boldsymbol{s_1 = 1.7288\ kJ/kg \cdot K}$$

State Point 3: Similarly the exit of the condenser is assumed to be a saturated liquid. From Table B.5 at $\boldsymbol{T = 40\ °C}$

$$\boldsymbol{h_3 = 256.54\ kJ/kg} \quad \text{and} \quad \boldsymbol{s_3 = 1.1909\ kJ/kg \cdot K}$$

State Point 2: If we assume that the compressor is isentropic then

$$\boldsymbol{s_1 = s_2 = 1.7288\ kJ/kg \cdot K}$$

Since the pressure at state point 2 is the same as at state point 3, we will need to find temperature and enthaply from Table B.5.2 through a double interpolation

at $\boldsymbol{P = 1000\ KPa}$ and $\boldsymbol{s = 1.7288\ kJ/kg \cdot K}$

$$\boldsymbol{h = 424.69\ kJ/kg} \qquad \text{and} \qquad \boldsymbol{T = 44.04\ °C}$$

at $\boldsymbol{P = 1200\ KPa}$ and $\boldsymbol{s = 1.7288\ kJ/kg \cdot K}$

$$\boldsymbol{h = 428.51\ kJ/kg} \qquad \text{and} \qquad \boldsymbol{T = 51.47\ °C}$$

Interpolate between these two values give

$$\boldsymbol{h_2 = 425.01\ kJ/kg} \qquad \text{and} \qquad \boldsymbol{T_2 = 44.7\ °C}$$

State Point 4: Assume that the throttling valve is adiabatic

$$\boldsymbol{h_3 = h_4 = 256.54\ kJ/kg}$$

Part a)
The rate of heat transfer in the evaporator and the condenser is given as

$$\boldsymbol{q_L = h_1 - h_4 = 395.34\ kJ/kg - 256.54\ kJ/kg = 138.8\ kJ/kg \Leftarrow Part\ a}$$

$$\boldsymbol{q_H = h_2 - h_3 = 425.01\ kJ/kg - 256.54\ kJ/kg = 168.47\ kJ/kg \Leftarrow Part\ a}$$

Part b)
The work of the compressor is given as

$$w_c = h_2 - h_1 = 425.01\ kJ/kg - 395.34\ kJ/kg = 29.67\ kJ/kg$$

$$COP = \frac{q_L}{w_c} = \frac{138.8\ kJ/kg}{29.67\ kJ/kg} = 4.68 \Leftarrow Part\ b$$

Part c)
The change in availability in the cold space is

$$\Delta\psi_L = (-q_L)\left(1 - \frac{T_0}{T_L}\right) = (-138.8\, kJ/kg)\left(1 - \frac{298.15}{-5 + 273.15}\right) = 15.53\, kJ/kg\ \Leftarrow\ Part\, c$$

Note: we use $-q_L$ because heat is leaving the cold space
The change in availability in the hot space is

$$\Delta\psi_H = (q_H)\left(1 - \frac{T_0}{T_H}\right) = (168.47\, kJ/kg)\left(1 - \frac{298.15}{40 + 273.15}\right) = 8.07\, kJ/kg \Leftarrow\ Part\, c$$

Note: we use q_L because heat is entering the hot space

Part d)
The second law efficiency is given as

$$\eta_{2nd} = \frac{\psi_L}{w_c} = \frac{15.53\ kJ/kg}{29.67\ kJ/kg} = 0.523\ \Leftarrow\ Part\ d$$

Part e)
The total cycle exergy destruction is given as

$$i = \frac{I}{\dot{m}} = T_o s_{gen}$$

where s_{gen} is obtained from an entropy balance on the system which yields

$$0 = \frac{q_L}{T_L} - \frac{q_H}{T_H} + s_{gen}$$

$$i = (298.15\ K)\left(\frac{168.47}{40 + 273.15}\right) - \left(\frac{138.8}{-5 + 273.15}\right) = 6.07\ kJ/kg$$

The exergy destruction is then given by

$$\dot{I} = \dot{m}i = \left(\frac{\dot{W}}{w_c}\right) i = \left(\frac{0.5\ kJ/s}{(425.01 - 395.34)\ kJ/kg}\right)\times 6.07\, kJ/kg = 0.1023\, kW \Leftarrow\ Part\, e$$