University of Waterloo

ME 354

THERMODYNAMICS 2
MIDTERM EXAMINATION

February 13, 2012 11:30 am - 1:30 pm

Instructor: R. Culham

Name: ______________________________

Student ID Number: ______________________________

Instructions

1. This is a 2 hour, closed-book examination.
2. Answer all questions in the space provided. If additional space is required, use the back of the pages or the blank pages included.
3. Permitted aids include:
   - Property Tables Booklet (Fundamentals of Thermodynamics, Borgnakke and Sonntag, 7th ed) or a photocopy of this booklet
   - one 8.5 in.× 11 in. crib sheet. (one side only)
   - calculator
4. It is your responsibility to write clearly and legibly. Clearly state all assumptions. Part marks will be given for part answers, provided that your methodology is clear.

Question	**Marks**	**Grade**
1	10	
2	20	
3	20	
TOTAL	50	

<u>**Question 1**</u> (10 marks)

Determine the maximum useful work in $\boldsymbol{kJ}$ that could be obtained from $\mathbf{0.6}\ \boldsymbol{m^3}$ of compressed air at $\mathbf{250}\ \boldsymbol{°C}$ and $\mathbf{700}\ \boldsymbol{kPa}$. Assume the dead state conditions to be $\boldsymbol{T_0 = 25\ °C}$ and $\boldsymbol{P_0 = 1\ atm\ = 101.325\ kPa}$.

Assumptions

1. air is an ideal gas
2. properties are independent of temperature and determined at $\mathbf{25}\ \boldsymbol{°C}$

The maximum useful work that could be obtained is equivalent to the availability of the air, $\boldsymbol{\Phi = m \times \phi}$

Air is assumed to be an ideal gas, therefore,

$$v = \frac{R \times T}{P} = \frac{(0.287\ kJ/kg \cdot K)(250 + 273.15)K}{(700\ kPa)\left(\dfrac{1\ kJ/m^3}{kPa}\right)} = 0.2145\ m^3/kg$$

The mass of the air is

$$m = \frac{V}{v} = \frac{0.6\ m^3}{0.2145\ m^3/kg} = 2.797\ kg$$

The specific control mass availability can be written as

$$\phi = (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$$

First,

$$(u - u_0) = c_v(T - T_0) = (0.717\ kJ/kg \cdot K)(523.15 - 298.15)K = 161.325\ kJ/kg$$

From the ideal gas equation used at the dead state

$$v_0 = \frac{R \cdot T_0}{P_0} = \frac{(0.287\ kJ/kg \cdot K)(298.15\ K)}{(101.325\ kPa)\left(\dfrac{1\ kJ/m^3}{kPa}\right)} = 0.8445\ m^3/kg$$

$$\begin{aligned} P_0(v - v_0) &= (101.325\ MPa)(0.2145 - 0.8445)m^3/kg\ \times\ \left(\frac{1\ kJ/m^3}{kPa}\right) \\ &= -63.835\ kJ/kg \end{aligned}$$

and

$$
\begin{aligned}
s - s_0 &= c_p \ln\left(\frac{T}{T_0}\right) - R \ln\left(\frac{P}{P_0}\right) \\
&= 1.004 \, \frac{kJ}{kg \cdot K} \ln\left(\frac{523.15}{298.15}\right) - (0.287) \, \frac{kJ}{kg \cdot K} \ln\left(\frac{700}{101.325}\right) \\
&= 0.0098 \; kJ/kg \cdot K
\end{aligned}
$$

and

$$
T_0(s - s_0) = (298.15 \; K)(0.0098 \; kJ/kg \cdot K) = 2.929 \; kJ/kg
$$

Collecting terms

$$
\begin{aligned}
\phi &= (u - u_0) + P_0(v - v_0) - T_0(s - s_0) \\
&= 161.325 - 63.835 - 2.929 = 94.561 \; kJ/kg
\end{aligned}
$$

The availability and in turn the maximum actual work that could be done is

$$
\Phi = m \times \phi = 2.797 \; kg \times 94.561 \; kJ/kg = 264.5 \; kJ \Leftarrow
$$

Question 2 (20 marks)

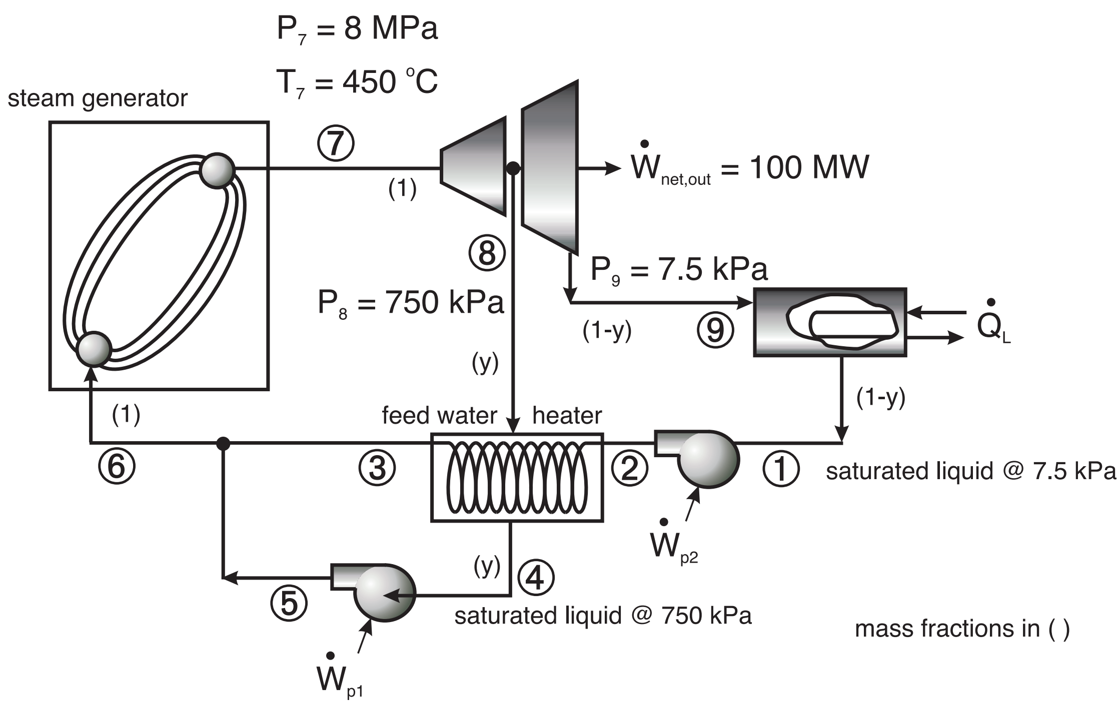
Water is used as the working fluid in a regenerative Rankine cycle where the circulation pumps and all stages of the turbine are considered to be isentropic. Superheated vapor enters the turbine at **8 MPa** and **450 °C**. After isentropic expansion in the first stage of the turbine, steam is extracted at an intermediate pressure of **0.75 MPa** and passed to a closed feedwater heater. The feedwater leaves the heater at **8 MPa** and a temperature equal to the saturation temperature at **0.75 MPa**. The saturated liquid condensate from the feedwater heater leaves at **0.75 MPa** and is pumped into the feedwater line as shown below. The condenser pressure is **7.5 kPa**. For a net power output from the cycle of **100 MW**.

a) draw the process diagram for this system and clearly label all relevant state points

b) determine the enthalpy at each state point in the process.

c) find the rate of heat transfer [MW] to the working fluid passing through the steam generator.

d) determine the thermal efficiency of the cycle.



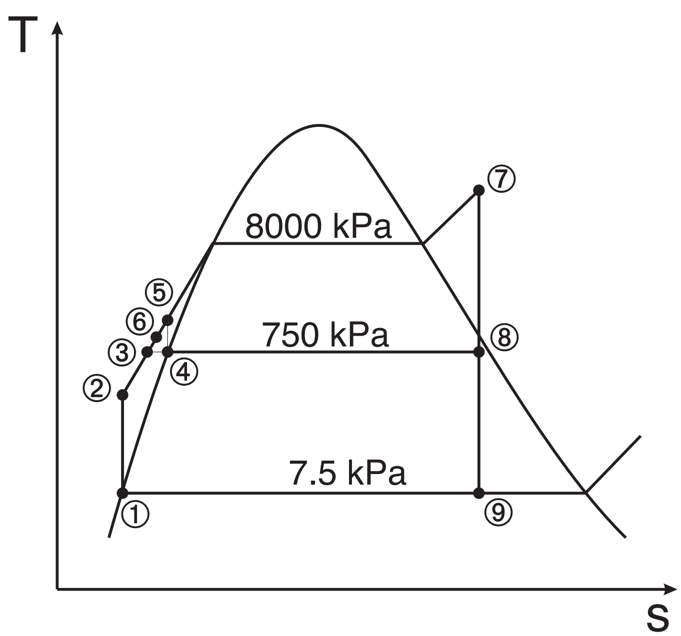
Assumptions

1. steady state
2. $\boldsymbol{KE = PE \approx 0}$
3. all processes are internally irreversible (except for the mixing of streams 3 and 5 to form 6)

4. turbines, pumps and feedwater heater are adiabatic
5. the condensate at the exit of the feedwater heat is a saturated liquid at **0.75 *MPa***
6. the condensate at the exit of the condenser is a saturated liquid at **7.5 *kPa***

State	P (kPa)	T $(°C)$	h (kJ/kg)	s $(kJ/kg \cdot K)$	v (m^2/kg)	
1	7.5		168.77		0.001008	sat. liq.
2	8000		176.83			
3	8000		713.34			sub cooled
4	750		709.45		0.001111	sat. liq.
5	8000		717.51			
6	8000		714.22			
7	8000	450	3271.99	6.5550		
8	750		2708.84	6.5550		$s_8 = s_7$
9	7.5		2043.00	6.5550		$s_9 = s_7$

Part a)



Part b)

State 8

$$x_8 = \frac{s_8 - s_f}{s_{fg}} = \frac{6.5550 - 2.0199}{4.6647} = 0.972$$

$$h_8 = x_8 \times h_{fg} + h_f = 0.972 \times 2056.98 + 709.45 = 2708.84 \ kJ/kg$$

State 9

$$x_9 = \frac{s_9 - s_f}{s_{fg}} = \frac{6.5550 - 0.5763}{7.6751} = 0.779$$

$$h_9 = x_9 \times h_{fg} + h_f = 0.779 \times 2406.02 + 168.77 = 2043.00\ kJ/kg$$

State 2

$$\begin{aligned} h_2 = h_1 + v_1(P_2 - P_1) &= 168.77\ \frac{kJ}{kg} + 0.001008\ \frac{m^3}{kg}(8000 - 7.5)\ kPa \left(\frac{1\ kJ/m^3}{kPa}\right) \\ &= 176.83\ kJ/kg \end{aligned}$$

State 5

$$\begin{aligned} h_5 = h_4 + v_4(P_5 - P_4) &= 709.45\ \frac{kJ}{kg} + 0.001111\ \frac{m^3}{kg}(8000 - 750)\ kPa \left(\frac{1\ kJ/m^3}{kPa}\right) \\ &= 717.51\ kJ/kg \end{aligned}$$

State 3

The liquid at point 3 is sub cooled and at a pressure of $8\ MPa$. We can determine the enthalpy by using a double iteration in Table B.1.4:
The saturation temperature at $0.75\ MPa$ is

$$T_{sat@0.75\ MPa} = 167.77\ ^\circ C$$

$$\text{at}\ \ 5\ MPa\ \ \text{and}\ \ 167.7\ ^\circ C \ \ \rightarrow\ \ h = 711.65\ kJ/kg$$

$$\text{at}\ \ 10\ MPa\ \ \text{and}\ \ 167.7\ ^\circ C \ \ \rightarrow\ \ h = 714.47\ kJ/kg$$

$$\text{at}\ \ 8.0\ MPa\ \ \text{and}\ \ 167.7\ ^\circ C \ \ \rightarrow\ \ h_3 = 713.34\ kJ/kg$$

State 6

The mass fraction, y, can be calculated by performing a heat balance over the feedwater heater

$$\begin{aligned} y \times h_8 + (1 - y) \times h_2 &= y \times h_4 + (1 - y)h_3 \\ y &= \frac{h_3 - h_2}{(h_8 - h_4) + (h_3 - h_2)} \\ &= \frac{713.34 - 176.83}{(2708.84 - 709.45) + (713.34 - 176.83)} = 0.2116 \end{aligned}$$

Using an energy balance at the common node between states 3, 5 and 6

$$\boldsymbol{h_6 = (1-y)h_3 + yh_5 = (1-0.2116)(713.34) + (0.2116)(717.51) = 714.22\ kJ/kg}$$

Part b)
The heat transfer in the steam generator is given as

$$\dot{Q}_H = \dot{m}_7(h_7 - h_6)$$

where the mass flow rate of the system can be determined as

$$\dot{m}_7 = \frac{\dot{W}_{net,out}}{W_{out}} = \frac{\dot{W}_{net,out}}{\dfrac{\dot{W}_t}{\dot{m}_7} - \dfrac{\dot{W}_p}{\dot{m}_7}}$$

The work in the turbine is

$$\begin{aligned}
\frac{\dot{W}_t}{\dot{m}_7} &= (h_7 - h_8) + (1-y)(h_8 - h_9) \\
&= (3271.99 - 2708.84) + (1-0.2116)(2708.84 - 2043.00)\ kJ/kg \\
&= 1088.1\ kJ/kg
\end{aligned}$$

The work of the pumps is

$$\begin{aligned}
\frac{\dot{W}_p}{\dot{m}_7} &= (1-y)(h_2 - h_1) + (y)(h_5 - h_4) \\
&= (1-0.2116)(176.83 - 168.77) + (0.2116)(717.51 - 709.45)\ kJ/kg \\
&= 8.06\ kJ/kg
\end{aligned}$$

and the mass flow in the system is

$$\dot{m}_7 = \frac{\dot{W}_{net,out}}{\dfrac{\dot{W}_t}{\dot{m}_7} - \dfrac{\dot{W}_p}{\dot{m}_7}} = \frac{(100\ MW)}{(1088.1 - 8.06)\ kJ/kg} \times \frac{1000\ kJ/s}{1\ MW} = 92.59\ kg/s$$

The heat transfer rate in the steam generator is

$$\begin{aligned}
\dot{Q}_{in} &= \dot{m}_7(h_7 - h_6) \\
&= (92.59\ kg/s)(3271.99 - 714.22)\ kJ/kg = \underline{236.8\ MW \Leftarrow}
\end{aligned}$$

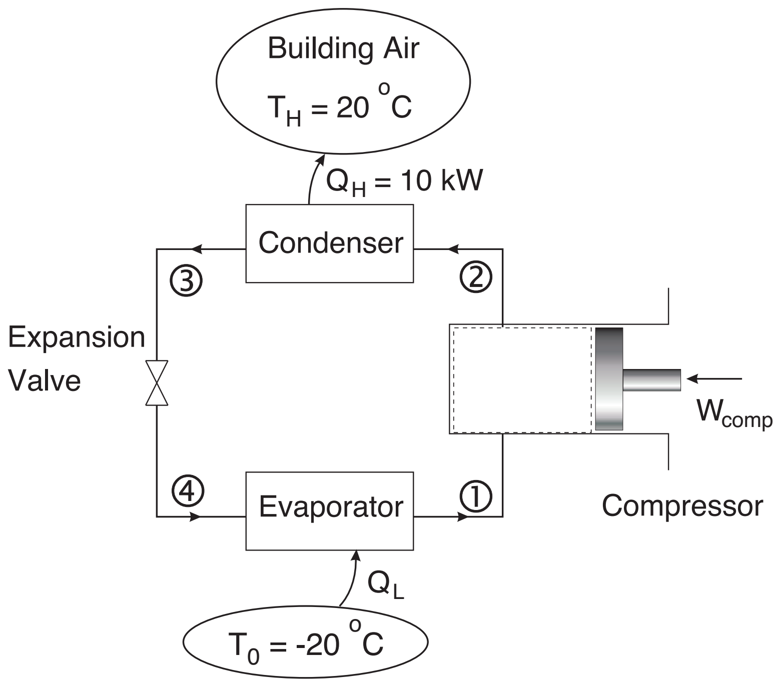
Part c)
The thermal efficiency is

$$\eta = \frac{\dot{W}_{out}}{\dot{Q}_{in}} = \frac{100\ MW}{236.8\ MW} = 0.422 = \underline{42.2\% \Leftarrow}$$

Question 3 (20 marks)

A vapour-compression, heat pump, using R-134a, is designed to provide $\boldsymbol{10\ kW}$ of heat between the reference atmosphere ($\boldsymbol{T_0}$) at $\boldsymbol{-20\ °C}$ and a building at $\boldsymbol{20\ °C}$. The inlet state to the compressor is a saturated vapour at $\boldsymbol{-30\ °C}$. The compressor exit state is at $\boldsymbol{50\ °C,\ 800\ kPa}$ while the inlet state to the valve is at $\boldsymbol{25\ °C}$.

a) determine the coefficient of performance of the heat pump
b) determine the availability destruction across each component in the system [$\boldsymbol{kJ/kg}$]
c) determine the second law efficiency
d) determine the mass flow rate [$\boldsymbol{kg/s}$]



State	$T[C]$	$P[kPa]$	$h[kJ/kg]$	$s[kJ/kg\cdot K]$
1	-30	85.1	379.80	1.7493
2	50	800	435.11	1.7768
3	25	800	234.7	1.1201
4	-30	85.1	234.7	1.1525

Assumptions

1. steady state, steady flow
2. $KE = PE = 0$
3. the condenser is considered a constant pressure device, therefore $P_2 = P_3$
4. since we do not have sub-cooled tables for R134a, we will use the property approximations in the sub-cooled region

Given that state point 3 is at $800\ kPa$ and $25\ °C$, this point is in the sub-cooled region. The approximations for sub-cooled properties give:

$$\begin{aligned} h_3(25\ °C) &= h_f(25\ °C) + v_f(25\ °C)[P_3 - P_{sat}(25\ °C)] \\ &= 234.59\ kJ/kg + 0.000829\ m^3/kg\,[800\ kPa - 666.3\ kPa]\,\frac{kJ}{kPa \cdot m^3} \\ &= 234.7\ kJ/kg \\ s_3(25\ °C) &= s_f(25\ °C) = 1.1201\ kJ/kg \cdot K \end{aligned}$$

To find the entropy at state point 4, we need to determine the quality of the refrigerant based on the value of the enthalpy.

$$h = h_f + x \cdot h_{fg}$$

or at $T_4 = -30\ °C$

$$x = \frac{234.7 - 161.12}{218.68} = 0.3365$$

Therefore the entropy is

$$s_4 = 0.8499(1 - 0.3365) + 0.3365(1.7493) = 1.1525\ kJ/kg \cdot K$$

Part a)

The COP of the heat pump is given as

$$COP = \frac{benefit}{cost} = \frac{\dot{Q}_{cond}/\dot{m}}{\dot{W}_{comp}/\dot{m}}$$

At the condenser

$$\frac{\dot{Q}_{cond}}{\dot{m}} = h_2 - h_3 = 435.11 - 234.7 = 200.41\ kJ/kg$$

At the compressor

$$\frac{\dot{W}_{comp}}{\dot{m}} = h_2 - h_1 = 435.11 - 379.80 = 55.31\ kJ/kg$$

And the COP is

$$COP = \frac{\dot{Q}_{cond}/\dot{m}}{\dot{W}_{comp}/\dot{m}} = \frac{200.41}{55.31} = 3.62 \Leftarrow$$

Part b)

The reference temperature is given as $T_0 = -20\ °C$.

At the compressor

$$\begin{aligned}\frac{\dot{I}_{comp}}{\dot{m}} &= T_0(s_2 - s_1) = (-20 + 273.15)\ K \times (1.7768 - 1.7493)\ kJ/kg \cdot K \\ &= 6.96\ kJ/kg\end{aligned}$$

At the condenser

$$\begin{aligned}\frac{\dot{I}_{cond}}{\dot{m}} &= T_0\left[(s_3 - s_2) + \frac{\dot{Q}_{cond}/\dot{m}}{T_H}\right] \\ &= (-20 + 273.15)\ K \times \left[(1.1201 - 1.7768) + \frac{200.41}{20 + 273.15}\right]\ kJ/kg \cdot K \\ &= 6.82\ kJ/kg\end{aligned}$$

At the expansion valve

$$\begin{aligned}\frac{\dot{I}_{valve}}{\dot{m}} &= T_0(s_4 - s_3) = (-20 + 273.15)\ K \times (1.1525 - 1.1201)\ kJ/kg \cdot K \\ &= 8.20\ kJ/kg\end{aligned}$$

At the evaporator

The heat transfer at the evaporator is given by

$$\frac{\dot{Q}_{evap}}{\dot{m}} = h_1 - h_4 = (379.80 - 234.7) = 145.1\ kJ/kg$$

$$\frac{\dot{I}_{evap}}{\dot{m}} = T_0\left[(s_1 - s_4) - \frac{\dot{Q}_{evap}/\dot{m}}{T_L}\right]$$

$$= (-20 + 273.15)\ K \times \left[(1.7493 - 1.1525) - \frac{145.1}{(-20 + 273.15)}\right]\ kJ/kg \cdot K$$

$$= 5.98\ kJ/kg$$

at T_L

For the heat transfer processes, we note that at the evaporator there is no entropy production because T_L is at the reference temperature T_0.

at T_H

$$\frac{\dot{I}_{T_H}}{\dot{m}} = \left[1 - \frac{T_0}{T_H}\right] \times \frac{\dot{Q}_{cond}}{\dot{m}} = \left[1 - \frac{253.15}{293.15}\right] \cdot 200.41\ kJ/kg = 27.35\ kJ/kg$$

Part c)
The second law efficiency is given as

$$\eta_{2nd} = \frac{\left[1 - \frac{T_0}{T_H}\right] \times \frac{\dot{Q}_{cond}}{\dot{m}}}{\dot{W}/\dot{m}} = \frac{27.35\ kJ/kg}{55.31\ kJ/kg} = 0.495 \ \Leftarrow$$

Part d)
The mass flow rate of the refrigerant is

$$\dot{m} = \frac{\dot{Q}_{cond}}{\dot{Q}_{cond}/\dot{m}} = \frac{10\ kW}{200.41\ kJ/kg} = 0.05\ kg/s \ \Leftarrow$$