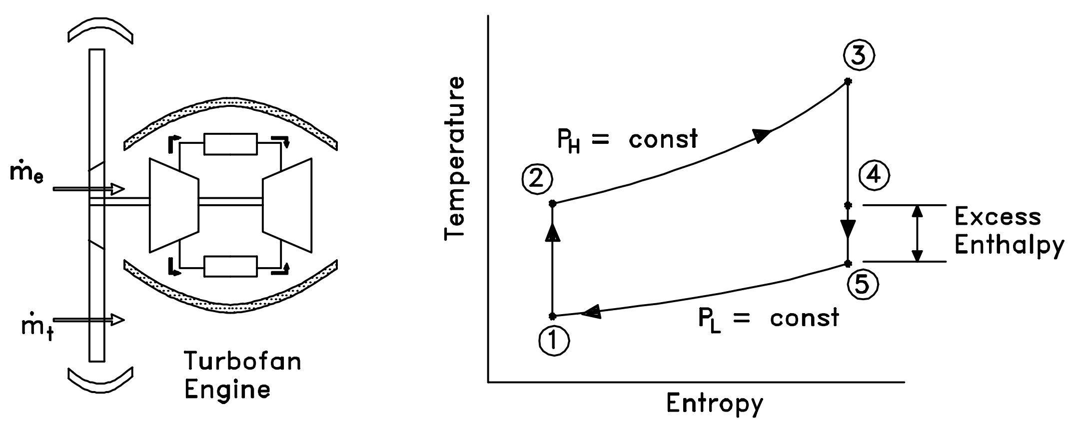
Problem: 7-5

Given: An aircraft with a turbofan propulsion system is flying at a speed of 200 m/s at an altitude where pressure is 60 kPa and temperature is 0 $°C$. The compressor has a pressure ratio of 8 : 1, the maximum Brayton cycle temperature is 400 $°C$, the area of the opening servicing the fan and engine is 3 m^2, and the mass flow rate ratio, defined as $r_m = (\dot{m}_e + \dot{m}_t)/\dot{m}_e$, is 4 : 1. Where $\dot{m}_e$ is the mass flow rate needed to supply the Brayton cycle engine and $\dot{m}_t$ is the thrust generating mass flow rate.

Find: The thrust force generated by this propulsion unit.



For the specified inlet conditions, the air's specific volume is

$$v = \frac{RT}{P} = 1.303\ m^3/kg$$

Air passes through the opening servicing the fan and compressor at the speed of the aircraft. The total mass flow rate is then

$$\dot{m} = \frac{AV}{v} = 460.5\ kg/s$$

of which

$$\dot{m}_e = \frac{\dot{m}}{r_m} = 115.1\ kg/s$$

passes through the Brayton cycle and

$$\dot{m}_t = \dot{m} - \dot{m}_e = 345.4\ kg/s$$

passes through the fan.
Following the state numbering scheme from the process diagram for the componenets in the Brayton cycle,the temperature at the end of the compression is

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = 495\ K$$

Similarly, the temperature at which the air leaves the engine is

$$T_5 = T_3 \left(\frac{P_5}{P_3}\right)^{(k-1)/k} = 371\ K$$

The temperature drop between states 3 and 4 is the same as that between states 1 and 2. Hence

$$T_4 = T_3 - (T_2 - T_1) = 451\ K$$

and the excess enthalpy is

$$h_4 - h_5 = c_p(T_4 - T_5) = 80.28\ kJ/kg$$

This excess enthalpy is converted into a kinetic energy change for the fan's thrust-producing stream. Equating the rate of kinetic energy production to the rate of excess enthalpy production (i.e. applying the first law) gives

$$\dot{m}_t \left(\frac{(v_e^*)^2 - (v_i^*)^2}{2}\right) = \dot{m}_e(h_4 - h_5)$$

When solved for the velocity at which the air leaves the fan, v_e^*, this yields

$$\begin{aligned} v_e^* &= \left(2\frac{\dot{m}_e}{\dot{m}_t}(h_4 - h_5) + (v_i^*)^2\right)^{1/2} \\ &= \left(2\ \times\ \frac{114.9}{344.5}\ \times\ 263.9\ \times\ 1000 + 200^2\right)^{1/2} \\ &= 306\ m/s \end{aligned}$$

The momentum thrust force produced by the fan is then

$$\begin{aligned} F_t &= \dot{m}_t(v_e^* - v_i^*) \\ &= \frac{344.5\ \times\ 106.0}{1000} \\ &= 36.52\ kN \end{aligned}$$