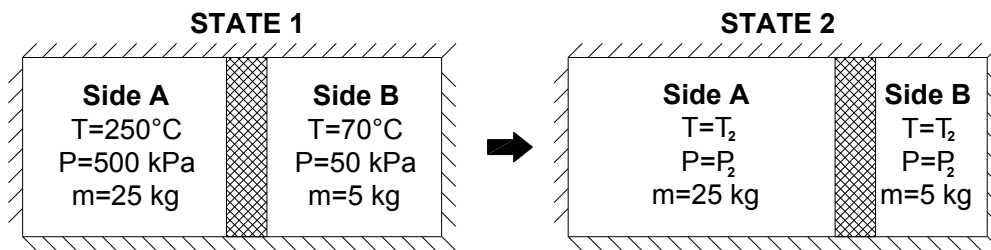


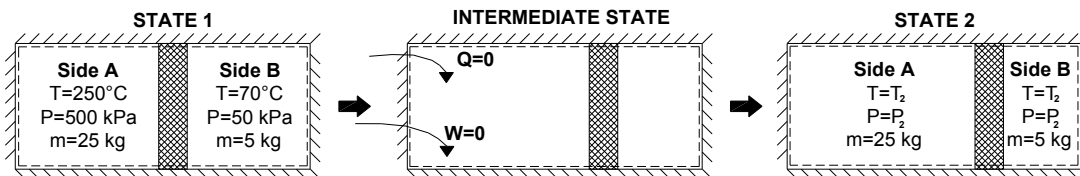
ME 354 Tutorial, Week#2 – Ideal Gas

Consider a rigid, insulated tank with a movable piston. Initially side A contains 25kg of air at 250°C and 500kPa, while side B contains 5kg of air at 70°C and 50kPa. The piston then moves to a new position where the pressures on each side will be equal. Since the piston is NOT adiabatic, the temperatures on each side will eventually equalize.

- a) Determine the final temperature
- b) Determine the final pressure



Step 1: Draw a diagram to represent the system showing control mass/volume of interest.



In this case, the control mass of interest (denoted by dashed lines) is the total system (both sides A & B). Each side could have been looked at separately in developing the solution as will be shown later.

Step 2: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

- a) T_2 – the final temperature of the system
- b) P_2 – the final pressure of the system

Step 3: Prepare a property table to keep track of the system's properties as you determine them

State	Property		
	T[K]	P[kPa]	m[kg]
A ₁	523	500	25
B ₁	343	50	5
A ₂	T_2	P_2	25
B ₂	T_2	P_2	5

Step 4: State your assumptions (you may have to add to your list of assumptions as you proceed in the problem)

Assumptions:

- 1) Rigid Tank – the air does NOT do work on the tank walls.
- 2) Insulated Tank – there is NO heat transfer from the tank to the surroundings.
- 3) Air is modelled as an ideal gas with constant specific heats over the temperature range of interest
- 4) Piston is NOT adiabatic so heat transfer can occur between sides A & B allowing for an equalization of temperature at state 2
- 5) $\Delta KE, \Delta PE \approx 0$

Part a)

Step 5: Calculations (usually start by writing first and second laws)

The First Law for a closed system can be expressed as Eq1.

$$E_1 + W_{1 \rightarrow 2} + Q_{1 \rightarrow 2} = E_2 \quad (\text{Eq1})$$

With our choice of the control mass boundary and the assumption of a rigid, insulated tank, the work and heat transfer terms in Eq1 will be zero, reducing Eq1 to Eq2 as shown below.

$$E_2 - E_1 = 0 \quad (\text{Eq2})$$

Recalling that $E = U + KE + PE$, Eq2 can be expanded into Eq3.

$$(U_2 - U_1) + (KE_2 - KE_1) + (PE_2 - PE_1) = 0 \quad (\text{Eq3})$$

With the assumption that ΔKE and ΔPE are approximately equal to zero, Eq3 reduces to Eq4.

$$U_2 - U_1 = 0 \quad (\text{Eq4})$$

The internal energy, U , of the control mass (total system) is composed of the internal energy of side A, U_A , and the internal energy of side B, U_B . Eq4 can be re-expressed in terms of the internal energies of side A & B at each state as shown in Eq5.

$$(U_{A2} + U_{B2}) - (U_{A1} + U_{B1}) = 0 \quad (\text{Eq5})$$

Rearranging and re-expressing Eq5 in terms specific internal energies (Recall $U = mu$), Eq6 is obtained.

$$m_A(u_{A2} - u_{A1}) + m_B(u_{B2} - u_{B1}) = 0 \quad (\text{Eq6})$$

Using the Ideal Gas Relation, $u_2 - u_1 = c_v(T_2 - T_1)$, Eq6 can be rewritten in terms of the system's temperatures as shown in Eq7.

$$m_A c_v (T_{A2} - T_{A1}) + m_B c_v (T_{B2} - T_{B1}) = 0 \quad (\text{Eq7})$$

As stated in the problem the piston is not adiabatic, so heat transfer will be permitted across it allowing for the temperatures of sides A & B to equalize. Introducing $T_{A2} = T_{B2} = T_2$ into Eq7 and isolating for T_2 , gives Eq8 – an equation for the final temperature of the system in terms of known quantities.

$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} \quad (\text{Eq8})$$

Substituting in values from the property table into Eq8 gives the solution for the final temperature of the system.

$$T_2 = \frac{(25)(523)[\text{kg} \cdot \text{K}] + (5)(343)[\text{kg} \cdot \text{K}]}{(25 + 5)[\text{kg}]} = \mathbf{493\text{K or } 220^\circ\text{C}} \quad \mathbf{\text{Answer a)}}$$

Part b)

To solve for the final pressure of the system we can make use of the Ideal Gas Law to express the pressure at state 2, P_2 , in two ways: the first in terms of the properties of side A, as shown in Eq9, the second in terms of the properties of side B, as shown in Eq10.

$$P_2 V_{A2} = m_A R T_2 \quad (\text{Eq9})$$

$$P_2 V_{B2} = m_B R T_2 \quad (\text{Eq10})$$

Adding Eq9 & Eq10, an expression for the pressure at state 2 is obtained in terms of the known quantities (m_A , m_B , R , T_2) as shown in Eq11. *The volumes of each side at state 2 (V_{A2} & V_{B2}) are still unknown.*

$$P_2 = \frac{m_A R T_2 + m_B R T_2}{V_{A2} + V_{B2}} \quad (\text{Eq11})$$

The denominator of Eq11, $V_{A2} + V_{B2}$, is equal to the total volume. The total volume of the system does not change during the process because we have assumed a rigid tank so, $V_{A2} + V_{B2} = V_{\text{Total}} = V_{A1} + V_{B1}$. Eq11 can be re-expressed in terms of V_{A1} & V_{B1} as shown in Eq12.

$$P_2 = \frac{m_A R T_2 + m_B R T_2}{V_{A1} + V_{B1}} \quad (\text{Eq12})$$

V_{A1} and V_{B1} can be determined from applying the Ideal Gas Law to side A and side B at state 1 as shown in Eq13 and Eq14.

$$V_{A1} = \frac{m_A R T_{A1}}{P_{A1}} \quad (\text{Eq13})$$

$$V_{B1} = \frac{m_B R T_{B1}}{P_{B1}} \quad (\text{Eq14})$$

Substituting in Eq13 and Eq14 into Eq12 for V_{A1} & V_{B1} , an expression, Eq15, for the system pressure at state 2 is found in terms of known quantities.

$$P_2 = \frac{m_A R T_2 + m_B R T_2}{\left(\frac{m_A R T_{A1}}{P_{A1}} \right) + \left(\frac{m_B R T_{B1}}{P_{B1}} \right)} \quad (\text{Eq15})$$

Dividing out the gas constant, R, in Eq15 gives Eq16

$$P_2 = \frac{(m_A + m_B) T_2}{\left(\frac{m_A T_{A1}}{P_{A1}} \right) + \left(\frac{m_B T_{B1}}{P_{B1}} \right)} \quad (\text{Eq16})$$

Substituting in values from the property table in Eq16 gives the solution for the final system pressure.

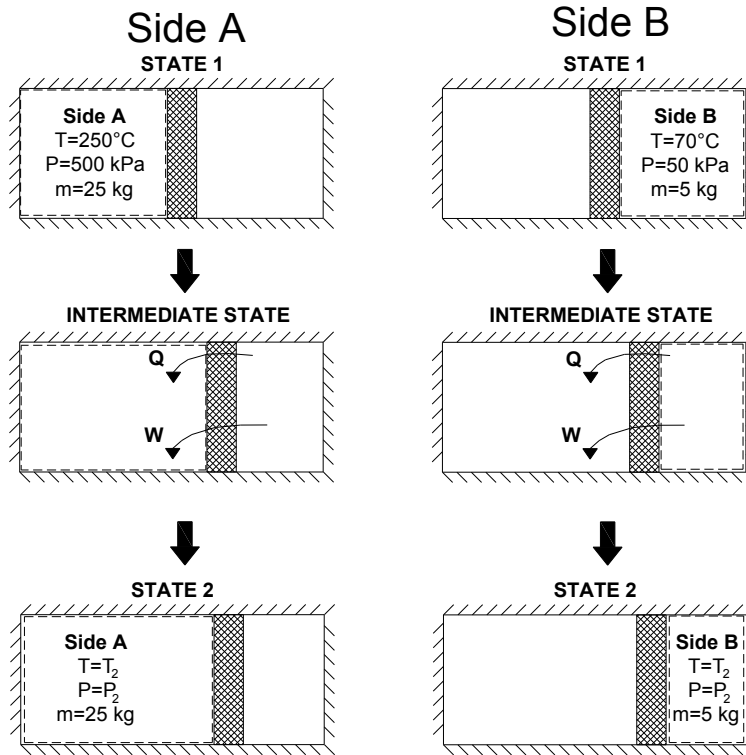
$$P_2 = \frac{(25 + 5)(493)[\text{kg} \cdot \text{K}]}{\frac{(25)(523)[\text{kg} \cdot \text{K}]}{500[\text{kPa}]} + \frac{(5)(343)[\text{kg} \cdot \text{K}]}{50[\text{kPa}]}} = 244.7 \text{ kPa} \quad \text{Answer b)}$$

Step 6: Concluding Statement

The final system temperature and pressure were found to be 220°C and 244.7 kPa respectively.

Alternative Approach to Part a)

Rather than choosing the control mass boundary to enclose the whole system, side A and B could have been analyzed separately. The diagram below shows the separate control masses defined in each case.



Writing the First Law equation for Side A and Side B gives Eq17 & Eq18

$$E_{A1} + W_{1 \rightarrow 2} + Q_{1 \rightarrow 2} = E_{A2} \quad (\text{Eq17})$$

$$E_{B1} - W_{1 \rightarrow 2} - Q_{1 \rightarrow 2} = E_{B2} \quad (\text{Eq18})$$

Combining Eq17 & Eq18 gives Eq19, which is equivalent to Eq2.

$$E_{A2} + E_{B2} - E_{A1} - E_{B1} = 0 \rightarrow E_2 - E_1 = 0 \quad (\text{Eq19})$$

The remainder of the solution can be obtained by following the same steps as before, starting at Eq2.