

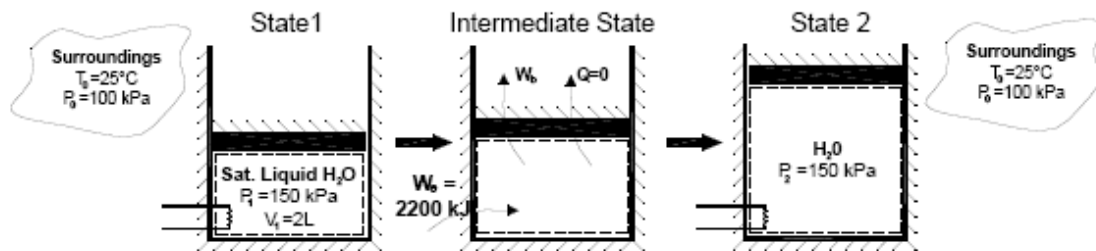
ME 354 – Tutorial, Week#4 – Exergy: Control Mass Analysis

An insulated piston-cylinder device contains 2L of saturated liquid water at a pressure of 150 kPa - which is constant throughout the process. An electric resistance heater inside the cylinder is turned on, and electrical work is done on the water in the amount of 2200 kJ. Assuming the surroundings to be at 25°C and 100 kPa, determine:

- The minimum work with which this process could be accomplished
- The exergy destroyed during this process

Step 1: Draw a diagram to represent the system (show control mass of interest)

The control mass boundary encloses the water in the cylinder



Step 2: Write out what is required to solve for

Find:

- The minimum work with which this process could be accomplished
- The entropy generated during the process

Step 3: Property table

State	Property					
	T[K]	P[kPa]	h[kJ/kg]	s[kJ/kg·K]	v[m ³ /kg]	V[m ³]
1(sat liq)		150				0.002
2		150				

Step 4: Assumptions

Assumptions:

- Insulated piston-cylinder (Q=0)
- $\Delta KE, \Delta PE \approx 0$
- Compression/expansion processes are in quasi-equilibrium

Step 5: Solve

Part a)

The solution for part a) involves finding the MINIMUM work with which this process could be accomplished. As stated in Cengel and Boles (Ch7), "Reversible Work, W_{rev} , is defined as... the minimum work that needs to be supplied as a system undergoes a process between the specified initial and final states." Therefore, the minimum work with which this process can

be accomplished is equal to the reversible work. Before the reversible work can be determined, the properties of the H₂O at the initial and final states must be determined. We can determine the properties at state 1 using the information given in the problem (saturated liquid H₂O at P = 150 kPa) with Table A-5.

From Table A-5

$$\rightarrow s_1 = s_{f@150\text{kPa}} = 1.4336 \text{ [kJ/kg}\cdot\text{K]}$$

$$\rightarrow v_1 = v_{f@150\text{kPa}} = 0.001053 \text{ [m}^3\text{/kg]}$$

$$\rightarrow h_1 = h_{f@150\text{kPa}} = 467.11 \text{ [kJ/kg]}$$

From the specific volume at state 1, v_1 , and the volume of state 1, V_1 , we can determine the mass of H₂O in the cylinder.

$$\rightarrow m_{H_2O} = \frac{V_1}{v_1} = \frac{0.002 \left[\frac{\text{m}^3}{\text{kg}} \right]}{0.001053 \left[\frac{\text{m}^3}{\text{kg}} \right]} = 1.9 \text{ [kg]}$$

The properties at state 2 must now be determined (the pressure at state 2 (150 kPa) is known because this is a constant pressure process). An energy balance on the system can be written to link state 2 to state 1, as shown in Eq1.

$$E_1 + W_e - W_b = E_2 \quad (\text{Eq1})$$

Using the assumption $\Delta KE, \Delta PE \approx 0$, and noting that $W_b = P(V_2 - V_1)$, Eq1 can be re-expressed as Eq2.

$$U_2 - U_1 = W_e - P(V_2 - V_1) \quad (\text{Eq2})$$

Eq2 can be rearranged to Eq3.

$$(U_2 + PV_2) - (U_1 + PV_1) = W_e \quad (\text{Eq3})$$

Recognizing that $U + PV = H$, and that $H = m_{H_2O}h$, Eq3 can be rewritten as Eq4.

$$m_{H_2O}(h_2 - h_1) = W_e \quad (\text{Eq4})$$

Isolating h_2 in Eq4, and substituting in the values for W_e , m_{H_2O} , and h_1 , h_2 can be determined.

$$\rightarrow h_2 = \frac{W_e}{m_{H_2O}} + h_1 = \frac{2200[kJ]}{1.9[kg]} + 467.11 \left[\frac{kJ}{kg} \right] = 1625 \left[\frac{kJ}{kg} \right]$$

Using $h_2 = 1625 \text{ kJ/kg}$ and $P_2 = 150 \text{ kPa}$ with Table A-5, we find that state 2 is within the vapor dome, in between the saturated liquid and vapor states on the 150 kPa isobar line. The properties at state 2 can determine by first finding the quality, and then using this to interpolate in Table A-5.

$$\rightarrow x = \frac{h_2 - h_f}{h_g - h_f} = \frac{1625 - 467.11}{2693.6 - 467.11} = 0.52$$

$$\rightarrow S_2 = S_{f@150\text{kPa}} + xS_{fg@150\text{kPa}} = 1.4336 + 0.52(5.7897) = 4.44 \text{ [kJ/kg}^*\text{K]}$$

$$\rightarrow V_2 = V_{f@150\text{kPa}} + x(V_{g@150\text{kPa}} - V_{f@150\text{kPa}}) = 0.001053 + 0.52(1.1582) = 0.6033 \text{ [m}^3\text{/kg]}$$

As mentioned previously, the minimum work that needs to be provided to accomplish this process represents the reversible work input, which can be computed from the difference in the system exergy at state 2 and state 1, as expressed in Eq5.

$$W_{rev} = X_2 - X_1 \quad (\text{Eq5})$$

The system of exergy at states 1 and 2 can be determined from the expressions in Eq6 and Eq7 respectively.

$$X_1 = (U_1 - U_0) - T_0(S_1 - S_0) + P_0(V_1 - V_0) \quad (\text{Eq6})$$

$$X_2 = (U_2 - U_0) - T_0(S_2 - S_0) + P_0(V_2 - V_0) \quad (\text{Eq7})$$

Substituting Eq6 and Eq7 into Eq5, an expression for the reversible work can be obtained as shown in Eq8.

$$W_{min} = W_{rev} = (U_2 - U_1) - T_0(S_2 - S_1) + P_0(V_2 - V_1) \quad (\text{Eq8})$$

Note how the dead state properties U_0 , S_0 , and V_0 cancel each other out in Eq8.

Substituting the previously obtained expression for internal energy represented in Eq2 into Eq8 and bringing the m_{H_2O} outside the bracket to convert the properties to specific properties, Eq8 can be written as Eq9.

$$W_{rev} = W_e - m_{H_2O} [(P - P_0)(v_2 - v_1) + T_0(s_2 - s_1)] \quad (\text{Eq9})$$

Substituting the known property values into Eq9, we can calculate the minimum work input.

$$\rightarrow W_{\min} = 2200[kJ] - 1.9[kg] \left[(150 - 100)(0.6033 - 0.001053) \left[\frac{kJ}{kg} \right] + (298)(4.44 - 1.4336) \left[\frac{kJ}{kg} \right] \right]$$

$$= 440.6 \text{ kJ} \quad \text{Answer a)}$$

Part b)

The exergy destroyed can be calculated from the expression shown in Eq10.

$$X_{\text{destroyed}} = T_0 S_{\text{gen}} \quad (\text{Eq10})$$

To find the entropy generated during the process we can perform an entropy balance on the system as shown in Eq11.

$$S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}} \quad (\text{Eq11})$$

For a closed system, entropy can only be carried across the system boundary by heat transfer. Since the system is insulated, $S_{\text{in}} = 0$ and $S_{\text{out}} = 0$. The change in the system entropy is the difference in entropy between state 2 and state 1 i.e. $\Delta S_{\text{system}} = m_{\text{H}_2\text{O}}(s_2 - s_1)$. Using this information with Eq11, the $X_{\text{destroyed}}$ (Irreversibility) can be determined from Eq12.

$$X_{\text{destroyed}} = T_0 m_{\text{H}_2\text{O}} (s_2 - s_1) \quad (\text{Eq12})$$

Substituting in the know values into Eq12, the $X_{\text{destroyed}}$ (Irreversibility) can be calculated.

\rightarrow

$$= 1702.2 \text{ kJ} \quad \text{Answer b)}$$

Part b) Other method

The exergy destroyed can also be calculated with an exergy balance.

$$\underbrace{X_{\text{in}} - X_{\text{out}}}_{\text{Exergy Transfer by Heat and Work}} - X_{\text{destroyed}} = X_2 - X_1 \quad (\text{Eq 13})$$

Exergy Transfer
by Heat and Work

The exergy change corresponds to the reversible work and was calculated in part a). The exergy entering the system corresponds to the electrical work.

$$X_{in} = W_e \quad (\text{Eq 14})$$

The exergy leaving the system can be calculated with the expression shown in Eq 15.

$$\begin{aligned} X_{out} &= W_{actual} - W_{sur} \\ X_{out} &= P_1(V_2 - V_1) - P_0(V_2 - V_1) \\ X_{out} &= (P_1 - P_0)(V_2 - V_1) \\ X_{out} &= (P_1 - P_0)(V_2 - V_1) \\ X_{out} &= m_{H_2O}(P_1 - P_0)(v_2 - v_1) \end{aligned} \quad (\text{Eq 15})$$

An expression for the exergy destroyed is obtained by subbing Eq(14) and Eq(15) in Eq(13).

$$X_{destroyed} = W_e - m_{H_2O}(P_1 - P_0)(v_2 - v_1) - (X_2 - X_1) \quad (\text{Eq 16})$$

Substituting the known values in Eq(16), we can calculate the exergy destroyed.

$$\begin{aligned} X_{destroyed} &= 2200[kJ] - 1.9[kg][150 - 100][kPa][0.6033 - 0.001053] \left[\frac{m^3}{kg} \right] - (440.6[kJ]) \\ &= 1702.2 \text{ kJ} \end{aligned}$$

Step 6: Concluding Statement and Remarks

The minimum work with which this process could be accomplished is 440.6kJ. The amount of exergy destroyed during the actual process is 1702.2 kJ. One may question why an input of 2200kJ is required in the actual process if the process requires a minimum of 440.6kJ to accomplish it with 1702.2kJ of exergy destroyed in the actual process (which sums to 2142.8 kJ). In the actual process we still have an excess amount of work out of the system above that which is required to expand against the surroundings pressure. This excess work can be expressed as $(P - P_0)m_{H_2O}(v_2 - v_1)$, which is equal to 57.2kJ. Summing these three components together accounts for the 2200kJ input.

Minimum Work Input =	440.6 kJ
+ Exergy Destroyed =	1702.2 kJ
+ Excess Work Output =	57.2 kJ
= Actual Work Input =	2200 kJ