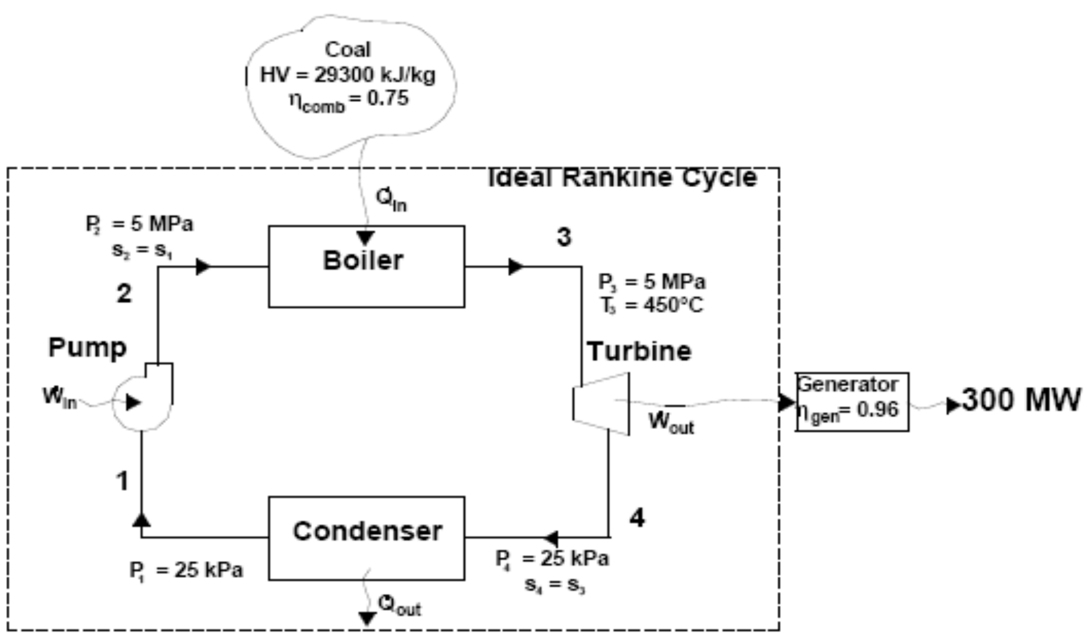
ME 354 – Tutorial, Week#5 – Rankine Cycle

Consider a coal-fired steam power plant that produces 300MW of electric power. The power plant operates on a simple *ideal* Rankine cycle with turbine inlet conditions of 5 MPa and 450°C and a condenser pressure of 25 kPa. The coal used has a heating value (energy released when the fuel is burned) of 29 300 kJ/kg. Assuming that 75% of this energy is transferred to the steam in the boiler and that the electric generator has an efficiency of 96%, determine:

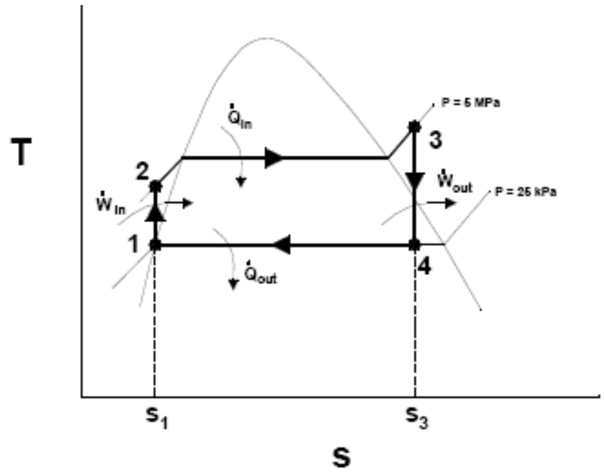
a) The overall plant efficiency (the ratio of net electric power output to the energy input as fuel)
b) The required rate of coal supply

Step 1: Draw a diagram to represent the system

The dashed line encloses the ideal Rankine Cycle. Since we are told this is an ideal Rankine Cycle we can assume isentropic compression ($s_2 = s_1$) and expansion ($s_4 = s_3$). We also know that boilers and condensers are modeled as constant pressure devices. Therefore $P_3 = P_2 = 5$ MPa & $P_4 = P_1 = 25$ kPa. Also, we can assume that state 1 is a saturated liquid @ $P_1 = 25$ kPa.



In order to understand the cycle better, it is always a good idea to draw a process diagram - in this case a T-s diagram.



Step 2: Prepare a property table

Preparing a property table becomes increasingly important when solving cycle-based problems.

	Property				
State	T [°C]	P [kPa]	h [kJ/kg]	s [kJ/kg*K]	v [m³/kg]
1 (sat liq)		25			
2		5000		s_1	v_1
3	450	5000			
4		25		s_3	

Step 3: Assumptions

Assumptions:

1) Steady operating conditions
2) Δke, Δpe $\approx$ 0
3) Isentropic compression (1$\rightarrow$ 2) / isentropic expansion (3$\rightarrow$ 4)
4) State 1 is a saturated liquid & incompressible substance
5) Boiler & condenser are constant pressure devices

Step 4: Calculations

Part a)

We are asked to find the overall plant efficiency of converting the chemical energy stored in the coal into electricity. Our overall efficiency will be based on:

1) How efficiently we can convert the energy stored in the coal into heat energy transferred into the boiler (η_{comb} = 0.75) as expressed in Eq1.

$$\dot{Q}_{in} = \eta_{comb}\,\dot{E}_{coal} \qquad \text{(Eq1)}$$

2) How efficiently the ideal Rankine Cycle operates in converting the heat input to the boiler into net work at the turbine shaft ($\eta_{th} = \dot{W}_{net,out} / \dot{Q}_{in}$) as expressed in Eq2.

$$\dot{W}_{net,out} = \eta_{th} \dot{Q}_{in} \qquad \text{(Eq2)}$$

3) How efficiently a generator can convert the net work output of the turbine to generate electricity ($\eta_{gen} = 0.96$) as expressed in Eq3.

$$\dot{E}_{elec} = \eta_{gen} \dot{W}_{net,out} \qquad \text{(Eq3)}$$

Substituting Eq1 into Eq2 we obtain Eq4.

$$\dot{W}_{net,out} = \eta_{comb} \eta_{th} \dot{E}_{coal} \qquad \text{(Eq4)}$$

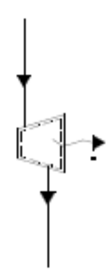
Substituting Eq4 into Eq3 we obtain Eq5.

$$\dot{E}_{elec} = \eta_{comb} \eta_{th} \eta_{gen} \dot{E}_{coal} \rightarrow \frac{\dot{E}_{elec}}{\dot{E}_{coal}} = \eta_{comb} \eta_{th} \eta_{gen} = \eta_{overall} \qquad \text{(Eq5)}$$

Since we know the values of the first and third efficiencies, the problem is reduced to finding the thermal efficiency of the cycle as expressed in Eq6.

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{out} - \dot{W}_{in}}{\dot{Q}_{in}} \qquad \text{(Eq6)}$$

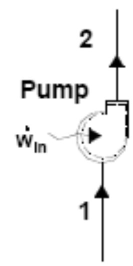
To find the rate of work output (turbine 3 $\rightarrow$ 4), we will draw a control volume that encloses the steam in the turbine as shown below.



Writing an energy balance on the turbine from state 3 → 4 using the assumptions that Δke, Δpe ≈ 0 and steady operating conditions exist we obtain Eq7.

$$\dot{W}_{out} = \dot{m}_{H2O}(h_3 - h_4) \qquad \text{(Eq7)}$$

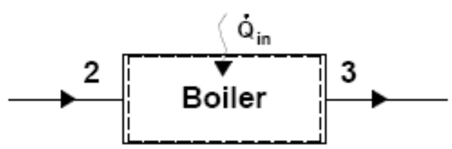
To find the rate of work input (pump 1 → 2), we will draw a control volume that encloses the water in the pump as shown below.



Writing an energy balance on the pump from state 1 → 2 using the assumptions that Δke, Δpe ≈ 0 and steady operating conditions exist we obtain Eq8.

$$\dot{W}_{in} = \dot{m}_{H2O}(h_2 - h_1) \qquad \text{(Eq8)}$$

To find the rate of heat input (boiler 2 → 3), we will draw a control volume that encloses the water/steam in the boiler as shown below.



Writing an energy balance on the boiler from state 2 → 3 using the assumptions that Δke, Δpe ≈ 0 and steady operating conditions exist we obtain Eq9.

$$\dot{Q}_{in} = \dot{m}_{H2O}(h_3 - h_2) \qquad \text{(Eq9)}$$

Substituting Eq7, Eq8, and Eq9 into Eq6 to obtain Eq10

$$\eta_{th} = \frac{\dot{W}_{out} - \dot{W}_{in}}{\dot{Q}_{in}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)} = 1 - \frac{q_{out}}{q_{in}} \qquad \text{(Eq10)}$$

Note: We could have just jumped from Eq6 to Eq10 and written the energy balances for the condenser and boiler.

Eq 10 shows that we must determine the enthalpy at each state point to calculate the thermal efficiency. We will start at state 1. Since we know that state 1 is saturated liquid H_2O @ P=25 kPa, we can use Table A-5 to determine the properties.

<u>State 1→ Table A-5 @ P=25 kPa</u>
$h_1=h_{f@P=25\ kPa}$ = 271.96 kJ/kg
$s_1=s_{f@P=25\ kPa}$ = 0.8932 kJ/kgK
$v_1= v_{f@P=25\ kPa}$= 0.001020 m^3/kg

<u>State 2</u>
To find the enthalpy of state 2, we express the reversible work in as shown in Eq 11. We treat the liquid H_2O as an incompressible substance ($v_1=v_2=v$)

$$\dot{W}_{in} = \dot{m}_{H_2O} v(P_2 - P_1) \qquad \text{(Eq11)}$$

Combining Eq11 with Eq8, and isolating for h_2 we obtain Eq12.

$$\dot{m}_{H_2O}(h_2 - h_1) = \dot{m}_{H_2O} v(P_2 - P_1) \rightarrow h_2 = v(P_2 - P_1) + h_1 \qquad \text{(Eq12)}$$

Substituting the known values into Eq12, we can determine h_2.

→h_2 = 0.001020 [m^3/kg] x (5000-25)[kN/m^2] + 271.96[kJ/kg] = 277 kJ/kg

Since we know the temperature (450°C) and pressure (5 MPa) at state 3, we can use the steam table to find the properties. Looking in Table A-5 @ P=5 MPa we see the corresponding saturated temperature is 263.94°C. Since the temperature is greater than the saturated temperature we must have superheated steam. We can make use of Table A-6 @ P=5 MPa & T=450°C to find the properties at state 3.

<u>State 3→ Table A-6 @ P=5 MPa, T=450°C</u>
h_3= 3317.2 kJ/kg
s_3= 6.8210 kJ/kgK

<u>State 4</u>
To find the enthalpy at state 4 we will use the fact that the expansion of the steam turbine from state 3→4 is isentropic.

→s_4=6.8210 kJ/kgK

Looking in Table A-5 using s_4 with the known pressure P_4 = 25 kPa, we see that s_4 is in between the saturated liquid value, s_f, and the saturated vapour value, s_g. We can now determine the quality of the saturated liquid-vapor mixture at state 4.

$$x_4 = \frac{s_4 - s_f}{s_g - s_f} = \frac{6.8210 - 0.8932}{7.8302 - 0.8932} = 0.855$$

The enthalpy at state 4 can be determined by interpolating using the quality between the saturated liquid and vapor states @ P=25 kPa in Table A-5.

Table A-5 @ P = 25 kPa
→ $h_4 = h_{f@P=25kPa} + xh_{fg@P=25kPa}$ = (271.96) + (0.855)(2345.5) = 2277.36 kJ/kg

Substituting the enthalpies into Eq10.

$$\rightarrow \eta_{th} = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{2277.36 - 271.96}{3317.2 - 277} = 0.34$$

Substituting this result into Eq5

$$\eta_{overall} = \eta_{comb}\eta_{th}\eta_{gen} = (0.75)(0.34)(0.96) = \textbf{24.5\% Answer a)}$$

Part b)
We need to find the required rate of coal supply. We are given that coal has a heating value of 29300 kJ/kg$_{coal}$ and that 75% of the energy from combusting the coal is transferred to the boiler. We can make use of the fact that the overall efficiency is the ratio of net electric power output, $\dot{E}_{elec}$, to the energy input as fuel, $\dot{E}_{coal}$, as was previously expressed in Eq5. The energy input as fuel can be expressed in terms of the heating value of coal and the rate of coal supply as shown in Eq13.

$$\dot{E}_{coal} = HV_{coal}\dot{m}_{coal} \qquad \text{(Eq13)}$$

Substituting Eq13 into Eq5, we obtain and expression for the rate of coal supply as shown in Eq 14.

$$\dot{m}_{coal} = \frac{\dot{E}_{electric}}{HV_{coal}\eta_{overall}} \qquad \text{(Eq14)}$$

Substituting in the electric power output (300 MW), the heating value of coal (29300 kJ/kg$_{coal}$), and the overall efficiency (0.245).

$$\dot{m}_{coal} = \frac{300000[kJ/s]}{29300[kJ/kg_{coal}](0.245)} = \textbf{41.79 kg}_{\textbf{coal}}\textbf{/s Answer b)}$$

Step 5: Concluding Statement and Remarks
The overall plant efficiency was found to be 24.5%. The required rate of coal supply was found to be 41.79 kg$_{coal}$/s.