

ME 354 Tutorial, Week#8 – IC Engines

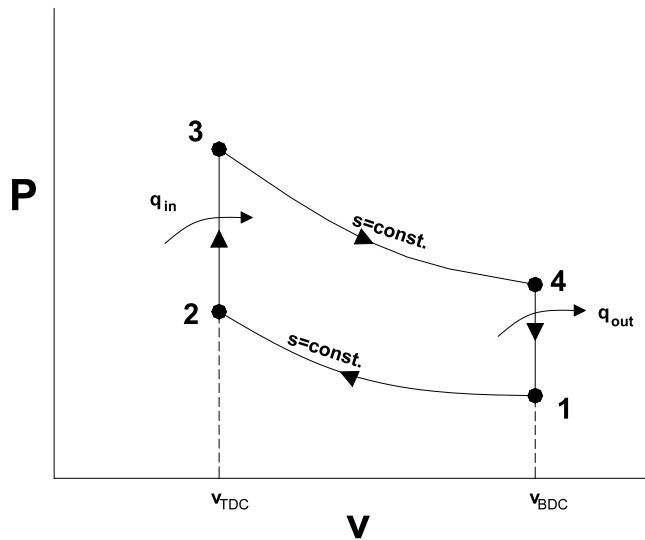
The compression ratio in an air-standard Otto cycle is 10. At the beginning of the compression stroke the pressure is 0.1 MPa and the temperature is 15°C. The heat transfer to the air per cycle is 1800 kJ/kg.

Determine:

- The pressure and temperature at the end of each process of the cycle,
- the net work output,
- the thermal efficiency,
- the mean effective pressure,
- and the irreversibility if this cycle was executed with a heat source temperature of 3500 K and a heat sink temperature of 250 K

Step 1: Draw a diagram to represent the system

A process diagram is drawn to visualize the processes occurring during the cycle.



Step 2: Write out what is required to solve for

- The pressure and temperature at the end of each process of the cycle
- the net work output
- the thermal efficiency
- the mean effective pressure
- the cycle irreversibility if this cycle was executed with a heat source temperature of 3500 K and a heat sink temperature of 250 K

Step 3: Property table

	T [K]	P [kPa]	v [m³/kg]
1	288	100	
2			
3			v ₂
4			v ₁

Step 4: Assumptions

- 1) $\Delta ke, \Delta pe \approx 0$
- 2) cold-air-standard assumption are applicable

Step 5: Solve**Part a)**

P_2 and T_2 will be determined first. Referring to the process diagram, state 1 to 2 is an isentropic compression process. Therefore the ideal gas relations for isentropic processes can be used. The temperature ratio of the two states is related to the specific volume ratio through k as shown in Eq1.

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{k-1} \quad (\text{Eq1})$$

Noting that the ratio v_1/v_2 (equivalent to V_{\max}/V_{\min}) is the compression ratio, r , and the value of k for air is 1.4, the temperature at state 2 can be determined.

$$T_2 = T_1 \left(\frac{v_1}{v_2} \right)^{k-1} = (288)(10)^{1.4-1} = 723.4[K] \quad \mathbf{T_2}$$

Again, since the process from state 1 to 2 is isentropic, the ideal gas relation relating the specific volume and pressure ratios through k can be used as shown in Eq2.

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2} \right)^k \rightarrow P_2 = P_1 \left(\frac{v_1}{v_2} \right)^k \quad (\text{Eq2})$$

Noting that v_1/v_2 is equal to the compression ratio, the pressure at state 2 can be determined as shown below.

$$P_2 = P_1 \left(\frac{v_1}{v_2} \right)^k = 100[kPa](10)^{1.4} = 2511.9[kPa] \quad \mathbf{P_2}$$

Performing an energy balance for the constant volume heat addition process (2 → 3), Eq3 is obtained.

$$q_{in} = u_3 - u_2 \quad (\text{Eq3})$$

For an ideal gas the internal energy is a function of temperature only. Using the assumption of constant specific heats evaluated at room temperature, the change in internal energy can be determined using Eq4.

$$u_3 - u_2 = c_v(T_3 - T_2) \quad (\text{Eq4})$$

The problem statement gives the value of q_{in} as 1800 kJ/kg. Substituting Eq4 into Eq3 along with the known value of q_{in} , the temperature at state 3 can be determined.

$$q_{in} = c_v(T_3 - T_2) \rightarrow T_3 = \frac{q_{in}}{c_v} + T_2 = \frac{1800 \left[\frac{kJ}{kg} \right]}{0.718 \left[\frac{kJ}{kg \cdot K} \right]} + 723.4K = 3230.4[K] \quad \mathbf{T_3}$$

Since the process from 2 to 3 is executed over a constant volume, the ideal gas law can be applied separately to both state 3 and state 2 and combined as shown below in Eq5.

$$v_2 = \frac{T_2 R}{P_2} = \frac{T_3 R}{P_3} = v_3 \rightarrow P_3 = P_2 \left(\frac{T_3}{T_2} \right) \quad (\text{Eq5})$$

Substituting the known values into Eq5, the pressure at state 3 can be solved for as shown below.

$$P_3 = P_2 \left(\frac{T_3}{T_2} \right) = (2511.9[kPa]) \left(\frac{3230.4K}{723.4K} \right) = 11216.6[kPa] \quad \mathbf{P_3}$$

Since the process from 3 to 4 is isentropic, the temperature at state 4 can be determined using the ideal gas relation relating the temperature and specific volume ratios through k as shown in Eq6.

$$T_4 = T_3 \left(\frac{v_3}{v_4} \right)^{k-1} \quad (\text{Eq6})$$

Noting that v_3/v_4 is the inverse of the compression ratio, the temperature at state 4 can be determined.

$$T_4 = (3230.4[K])\left(\frac{1}{10}\right)^{1.4-1} = 1286[K] \quad \mathbf{T_4}$$

The pressure can be determined from the isentropic relation for an ideal gas, which relates the pressure and the specific volume ratios through k as shown in Eq7.

$$P_4 = P_3 \left(\frac{v_3}{v_4} \right)^k \quad (\text{Eq7})$$

Noting again that v_3/v_4 is the inverse of the compression ratio, the pressure at state 4 can be determined.

$$P_4 = P_3 \left(\frac{v_3}{v_4} \right)^k = 11216.6[kPa] \left(\frac{1}{10} \right)^{1.4} = 445.6[kPa] \quad \mathbf{P_4}$$

Part b)

An overall energy balance on the cycle can be used to find an expression for the net work output as shown in Eq8.

$$q_{in} + w_{in} = q_{out} + w_{out} \rightarrow w_{net} = w_{out} - w_{in} = q_{in} - q_{out} \quad (\text{Eq8})$$

The value of q_{in} is given in the problem statement so the problem reduces to finding the value of q_{out} . Performing an energy balance for the process from 4 to 1, q_{out} can be determined from the temperature difference between state 4 and 1 as shown in Eq9.

$$q_{out} = u_4 - u_1 = c_v (T_4 - T_1) \quad (\text{Eq9})$$

Substituting the known values into Eq9, q_{out} can be determined as shown below.

$$q_{out} = c_v (T_4 - T_1) = \left(0.718 \left[\frac{kJ}{kg \cdot K} \right] \right) (1286[K] - 288[K]) = 716.6 \left[\frac{kJ}{kg} \right]$$

Using this result with the given $q_{in} = 1800 \text{ kJ/kg}$ and Eq8, the net work output can be determined as shown below.

$$w_{net} = q_{in} - q_{out} = (1800 - 716.6) \left[\frac{kJ}{kg} \right] = 1083.4 \left[\frac{kJ}{kg} \right]$$

**Answer
(b)**

Part c)

To calculate the thermal efficiency the general expression for efficiency (benefit/cost) can be used.

$$\eta_{th} = \frac{\text{benefit}}{\text{cost}} = \frac{w_{net}}{q_{in}} = \frac{1083.4[kJ]}{1800[kJ]} = 60.2\%$$

**Answer
(c)**

The Otto cycle thermal efficiency can also be determined using the equation that makes use of the compression ratio.

$$\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}} = 1 - \frac{1}{10^{0.4}} = 60.2\%$$

**Answer
(c)**

Part d)

The mean effective pressure (MEP) can be determined using Eq10.

$$MEP = \frac{w_{net}}{v_1 - v_2} \quad (\text{Eq10})$$

The value of w_{net} was determined in part b) but the values v_1 and v_2 are unknown. v_1 can be determined by applying the ideal gas law to state 1 as shown below.

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(0.287 \left[\frac{kJ}{kg \cdot K} \right] \right) (288[K])}{100[kPa]} = 0.827 \left[\frac{m^3}{kg} \right]$$

v_2 is related v_1 through the compression ration, r , and can be determined as shown below.

$$\frac{v_1}{v_2} = r \rightarrow v_2 = \frac{v_1}{r} = \frac{0.827 \left[\frac{m^3}{kg} \right]}{10} = 0.0827 \left[\frac{m^3}{kg} \right]$$

Substituting these results into Eq10, the value of the MEP can be determined as shown below.

$$MEP = \frac{w_{net}}{v_1 - v_2} = \frac{1083.4 \left[\frac{kJ}{kg} \right]}{(0.827 - 0.827) \left[\frac{m^3}{kg} \right]} = 1456.4 [kPa]$$

**Answer
(d)**

Part e)

The irreversibility of the cycle (exergy destroyed) if the source and sink temperatures were 3500 K and 250 K respectively, can be determined from application of Eq11.

$$x_{destroyed} = T_0 s_{gen} \quad (Eq11)$$

The entropy generated during this cycle can be determined by performing an entropy balance over each process as shown in Eq12 - 15.

Since the process from 1 to 2 is isentropic with no heat transfer and occurs in a closed system there will be no entropy generated.

$$s_{gen,1 \rightarrow 2} = \Delta s_{sys} + s_{out} - s_{in} = 0 \quad (Eq12)$$

Since the process from 2 to 3 occurs over constant volume with heat transfer into the system, there will be entropy generated as shown in Eq13.

$$s_{gen,2 \rightarrow 3} = \Delta s_{sys} + s_{out} - s_{in} = (s_3 - s_2) - \frac{q_{in}}{T_{source}} \quad (Eq13)$$

Since the process from 3 to 4 is isentropic with no heat transfer and occurs in a closed system there will be no entropy generated.

$$s_{gen,3 \rightarrow 4} = \Delta s_{sys} + s_{out} - s_{in} = 0 \quad (Eq14)$$

Since the process from 4 to 1 occurs over constant volume with heat transfer out of the system, there will be entropy generated as shown in Eq15.

$$s_{gen,4 \rightarrow 1} = \Delta s_{sys} + s_{out} - s_{in} = (s_1 - s_4) + \frac{q_{out}}{T_{sink}} \quad (Eq15)$$

The total entropy generated will be the sum of the entropy generated during each process as shown in Eq16.

$$s_{gen} = \left(\frac{q_{out}}{T_{sink}} - \frac{q_{in}}{T_{source}} \right) + (s_1 - s_4) + (s_3 - s_2) \quad (\text{Eq16})$$

Since the compression and expansion processes are modeled as isentropic $s_4 = s_3$ and $s_2 = s_1$. Therefore Eq16 reduces to Eq17.

$$s_{gen} = \left(\frac{q_{out}}{T_{sink}} - \frac{q_{in}}{T_{source}} \right) \quad (\text{Eq17})$$

The total entropy generated during the cycle is determined by substituting all of the known parameters into Eq17 as shown below.

$$s_{gen} = \left(\frac{716.6 \left[\frac{kJ}{kg} \right]}{250[K]} - \frac{1800 \left[\frac{kJ}{kg} \right]}{3500[K]} \right) = 2.352 \left[\frac{kJ}{kg \cdot K} \right]$$

Substituting this result into Eq11, the irreversibility of cycle is determined as shown below.

$$x_{destroyed} = T_0 s_{gen} = (298[K]) \left(2.352 \left[\frac{kJ}{kg \cdot K} \right] \right) = 700.93 \left[\frac{kJ}{kg} \right] \quad \textbf{Answer (e)}$$

Step 5: Concluding Remarks & Discussion

The pressures and temperatures at the end of each process are summarized in the table below.

	T [K]	P [kPa]
1	288	100
2	723.4	2511.9
3	3230.4	11216.6
4	1286	445.6

The net work output was found to be 1083.4 kJ/kg. The thermal efficiency of the cycle was found to be 60.2%. The MEP was determined to be 1456.4 kPa. The irreversibility of the cycle if the source and sink temperatures were 3500 K and 250 K would be 700.93 kJ/kg.