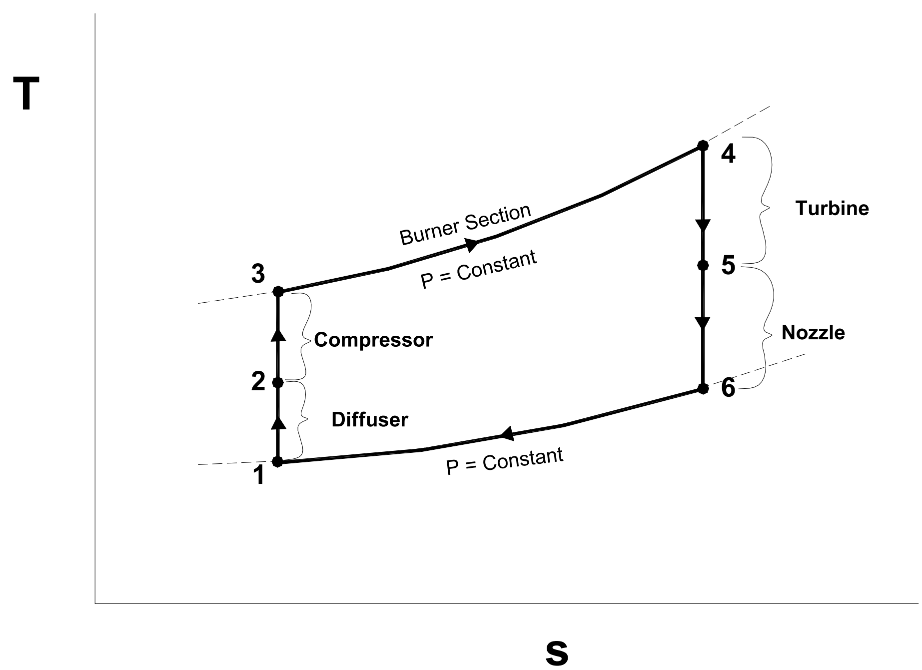
ME 354 Tutorial, Week #10
Jet-Propulsion Cycle

A turbojet is flying with a velocity of 320 m/s at an altitude of 9150m, where the ambient conditions are 32 kPa and -32°C. The pressure ratio across the compressor is 12, and the temperature at the turbine inlet is 1400 K. Air enters the compressor at a rate of 40 kg/s, and the jet fuel has a heating value of 42,700 kJ/kg. Assuming ideal operation for all components and constant specific heats for air at room temperature, determine:

a) the temperature and pressure at the turbine exit,
b) the velocity of the exhaust gases,
c) the propulsive power developed,
d) the propulsive efficiency, and
e) the rate of fuel consumption.

Step 1: Draw a diagram to represent the system

To better visualize what is happening during the cycle we can draw a T-s process diagram.



Step 2: Prepare a property table

	T [K]	P [kPa]
1	241	32
2		
3		12 P_2
4	1400	P_3
5		
6		32

Step 3: State your assumptions

Assumptions:

1) $\Delta pe \cong 0$ for all components
2) $\Delta ke \cong 0$ for compressor, burner, and turbine sections ONLY
3) cold-air-standard assumptions are applicable
4) ideal & steady operation of all components
5) $w_{comp,in} = w_{turb,out}$

Step 4: Solve

Part a)

In order to determine the temperature at the turbine exit an energy balance over the turbine can be performed as shown in Eq1. Note: the assumptions of steady operating conditions and Δke, $\Delta pe \cong 0$ are used in the energy balances of Eq1 and Eq3.

$$w_{turb,out} = (h_4 - h_5) \qquad \text{(Eq1)}$$

Since the air has been modeled as an ideal gas with constant specific heats at room temperature, Eq1 can be written as Eq2.

$$w_{turb,out} = (h_4 - h_5) = c_p(T_4 - T_5) \qquad \text{(Eq2)}$$

Similarly, performing an energy balance for the compressor, Eq3 is obtained.

$$w_{comp,in} = (h_3 - h_2) = c_p(T_3 - T_2) \qquad \text{(Eq3)}$$

Since the work produced by the turbine is assumed to be equal to the work supplied to the compressor, Eq2 can be equated to Eq3, as shown in Eq4.

$$w_{turb,out} = w_{comp,in} \rightarrow c_p(T_4 - T_5) = c_p(T_3 - T_2) \qquad \text{(Eq4)}$$

Isolating for T_5 in Eq4, Eq5 is obtained.

$$T_5 = T_4 + T_2 - T_3 \qquad \text{(Eq5)}$$

Since T_4 is given in the problem statement, only T_2 and T_3 must be determined to solve for T_5. Performing an energy balance on the diffuser, Eq6 is obtained.

$$\frac{dE_{cv}}{dt}$$

$$\frac{d\dot{E}_{cv}}{dt} = \dot{m}_{air}\left[(ke + pe + u + Pv)_1 - (ke + pe + u + Pv)_2\right] \qquad \text{(Eq6)}$$

$$\dot{E}_{cv}$$

Using the steady operation assumption, the assumption that $\Delta pe \cong 0$, and the definition of enthalpy ($h = u + Pv$), Eq6 reduces to Eq7. For a diffuser the outlet velocity will be negligible compared to the inlet velocity. Therefore, it can be assumed that $V_2 \cong 0$ and consequently, $ke_2 \cong 0$.

$$ke_1 + h_1 - h_2 = 0 \rightarrow c_p(T_2 - T_1) = \frac{V_1^2}{2} \rightarrow T_2 = \frac{V_1^2}{2c_p} + T_1 \qquad \text{(Eq7)}$$

Assuming the jet is flying in still air, the velocity of the air (relative to the jet) at the inlet of the diffuser is equal to the velocity of the jet. Therefore, V_1 = 320 m/s. From Table A-2 for air at room temperature c_p = 1.005 kJ/kg*K. Substituting these values into Eq7, the temperature at location 2 can be solved for as shown below.

$$T_2 = \frac{V_1^2}{2c_p} + T_1 = \frac{(320)^2\left[\frac{m^2}{s^2}\right]}{2(1.005)\left[\frac{kJ}{kg \bullet K}\right]} + 241[K] = \frac{102400\left[\frac{m^2}{s^2}\right]}{2.01(10^3)\left[\frac{m^2}{s^2 \bullet K}\right]} + 241[K]$$

$$\rightarrow T_2 = 291.9[K]$$

The pressure at location 2 can be determined by noting that the diffuser process is modeled as isentropic. This allows the use of the ideal gas relation for isentropic processes as shown below. From Table A-2 for air at room temperature k = 1.4.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \rightarrow \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} \rightarrow P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = (32[kPa])\left(\frac{291.9[K]}{241[K]}\right)^{\frac{1.4}{0.4}}$$

$$\rightarrow P_2 = 62.6[kPa]$$

Using the compressor pressure ratio of 12 (given in the problem statement), the pressure at location 3 can be solved for as shown below.

$$P_3 = 12P_2 = 12(62.6[kPa])$$
$$\rightarrow P_3 = 751.2[kPa]$$

Since the compression process is modeled as isentropic, the temperature at the compressor exit, T_3, can be solved using the ideal gas relation for isentropic processes as shown below.

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{k-1}{k}} \rightarrow T_3 = T_2\left(\frac{P_3}{P_2}\right)^{\frac{k-1}{k}} = (291.9[K])(12)^{\frac{0.4}{1.4}}$$
$$\rightarrow T_3 = 593.7[K]$$

Having determined T_2 and T_3, Eq5 can now be used to determine the temperature at the turbine exit as shown below.

$$T_5 = T_4 + T_2 - T_3 = 1400[K] + 291.9[K] - 593.7[K]$$
$$\rightarrow T_5 = 1098.2[K]$$

Answer a)

The pressure at the turbine exit can be determined using the fact that the expansion process through the turbine is modeled as isentropic. The ideal gas relation for isentropic processes can be applied as shown in Eq8.

$$\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}} \rightarrow \frac{P_5}{P_4} = \left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}} \rightarrow P_5 = P_4\left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}} \quad \text{(Eq8)}$$

The pressure across the burner section (3 → 4) is constant so $P_4 = P_3 =$ 751.2 kPa. Also, T_4 is given in the problem statement and T_5 was determined in the previous step. Substituting these values into Eq8, P_5 can be solved for as shown below.

$$P_5 = (751.2[kPa])\left(\frac{1098.2[K]}{1400[K]}\right)^{\frac{1.4}{0.4}}$$
$$\rightarrow P_5 = 321.1[kPa]$$

Answer a)

$$\frac{d\dot{E}_{cv}}{dt}$$

$$\dot{E}_{cv}$$

Part b)
To find the velocity of the exhaust gases, an energy balance can be performed on the nozzle (5 → 6) as shown in Eq9.

$$\frac{d\dot{E}_{cv}}{dt} = \dot{m}_{air}\left[(ke + pe + u + Pv)_5 - (ke + pe + u + Pv)_6\right] \qquad \text{(Eq9)}$$

Using the steady operation ass$\frac{d\dot{E}_{cv}}{dt} = \dot{m}_{air}\left[(ke + pe + u + Pv)_5 - (ke + pe + u + Pv)_6\right]$umption, the assumption that Δpe ≅ 0, and the definition of enthalpy (h = u + Pv), Eq9 reduces to Eq10. For nozzle analysis it is reasonable to assume that the inlet velocity is negligible compared to the exit velocity. Therefore it can be assumed that $V_5 \cong 0$ and consequently, $ke_5 = 0$.

$$h_5 - ke_6 - h_6 = 0 \rightarrow \frac{V_6^2}{2} = c_p(T_5 - T_6) \rightarrow V_6 = \sqrt{2c_p(T_5 - T_6)} \qquad \text{(Eq10)}$$

From Eq10, it is observed that the temperature at the exit of the nozzle (Location 6) must first be determined.

Since the process through the nozzle is modeled as isentropic, the ideal gas relation for isentropic processes can be used as shown below. The pressure at location 6 is ambient (32 kPa).

$$\frac{T_6}{T_5} = \left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} \rightarrow T_6 = T_5\left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = (1098.2[K])\left(\frac{32[kPa]}{321.1[kPa]}\right)^{\frac{0.4}{1.4}}$$
$$\rightarrow T_6 = 568.2[K]$$

Substituting this result into Eq10, the velocity of the exhaust gases can be determined as shown below.

$$\rightarrow V_6 = \sqrt{2\left(1.005(10^3)\left[\frac{J}{kg \bullet K}\right]\right)(1098.2[K] - 568.2[K])}$$
$$\rightarrow V_6 = \sqrt{1065300\left[\frac{m^2}{s^2}\right]}$$
$$\rightarrow V_6 = 1032[m/s]$$

Answer b)

Part c)

The propulsive power developed is defined as shown in Eq11.

$$\dot{W}_p = \dot{m}_{air}(V_{exit} - V_{inlet})V_{aircraft} \qquad \text{(Eq11)}$$

Substituting in the known values into Eq11, the propulsive power developed can be determined as shown below. Note: it was previously noted that $V_{inlet} = V_{aircraft}$.

$$\dot{W}_p = \left(40\left[\frac{kg}{s}\right]\right)\left(1032\frac{m}{s} - 320\frac{m}{s}\right)\left(320\frac{m}{s}\right) = 9113600\left[\frac{kg \bullet m \bullet m}{s^2 \bullet s}\right]$$

Answer c)

$$\rightarrow \dot{W}_p = 9113.6[kW]$$

Part d)

The propulsive efficiency is defined as the propulsive power developed divided by the rate of heat input as shown in Eq12.

$$\eta_p = \frac{\dot{W}_p}{\dot{Q}_{in}} \qquad \text{(Eq12)}$$

The propulsive power developed was determined in part c). The rate of heat input can be determined from an energy balance on the burner section (3 → 4) as shown in Eq13.

$$\dot{Q}_{in} = \dot{m}_{air}(h_4 - h_3) = \dot{m}_{air}\, c_p (T_4 - T_3) \qquad \text{(Eq13)}$$

Substituting in the known values into Eq13 the rate of heat input can be determined.

$$\dot{Q}_{in} = \left(40\frac{kg}{s}\right)\left(1.005\left[\frac{kJ}{kg \bullet K}\right]\right)(1400[K] - 593.7[K])$$

$$\rightarrow \dot{Q}_{in} = 32413.3[kW]$$

Substituting in the known values into Eq12, the propulsive efficiency can be determined as shown below.

$$\rightarrow \eta_p = \frac{\dot{W}_p}{\dot{Q}_{in}} = \frac{9113.6[kW]}{32413.3[kW]} = 0.2812$$

Answer d)

Part e)

The rate of fuel consumption can be determined from the heating value of fuel and the rate of heat input as shown below.

$$\dot{m}_{fuel} = \frac{\dot{Q}_{in}}{HV} = \frac{32413.3\frac{kJ}{s}}{42700\frac{kJ}{kg}} = 0.759[kg/s] \qquad \textbf{Answer e)}$$

Step 5: Concluding Remarks & Discussion

1) The temperature and pressure at the turbine exit are 1098.2 K & 321.1 kPa respectively.
2) The exhaust gas velocity is 1032 m/s.
3) The propulsive power developed is 9113.6 kW.
4) The propulsive efficiency is 28.12%.
5) The rate of fuel consumption is 0.759 kg/s.

Let us analyze,