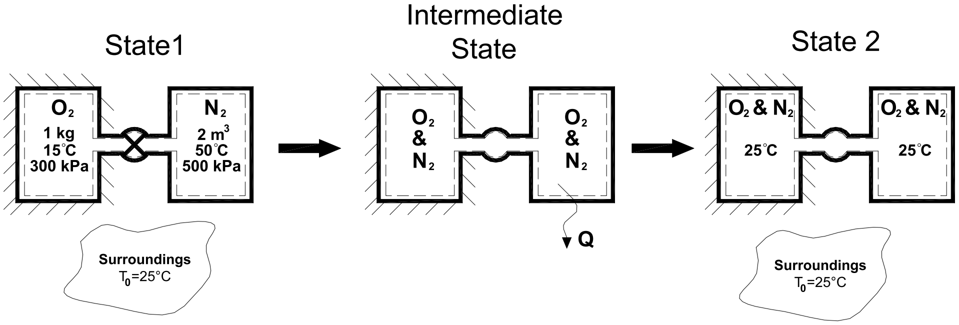
ME 354 Tutorial #8
Non-Reacting Mixtures – Ideal Gas Mixtures

An insulated tank that contains 1kg of O_2 at 15°C and 300kPa is connected to a 2m^3 un-insulated tank that contains N_2 at 50°C and 500kPa. The valve connecting the two tanks is opened, and the two gases form a homogeneous mixture at 25°C. Assume the surroundings to be at 25°C.

Determine:

a) the final pressure in the tank,
b) the heat transfer,
c) the entropy generated during this process, and
d) the exergy destruction associated with this process.

Step 1: Draw a diagram to represent the system



Step 2: Prepare a property table

	T (K)	P (kPa)	V (m^3)	m (kg)
O_2, 1	288	300		1
N_2, 1	323	500	2	
$(O_2 + N_2)$,2	298		V_1+V_2	m_1+m_2

Step 3: State your assumptions

Assumptions:

1) O_2, N_2, and mixture are modeled as ideal gases.
2) ΔKE, ΔPE ≈ 0
3) Tanks are rigid $\therefore$W = 0.

Step 4: Calculations

Part a)

To find the total pressure in the tank after mixing we can make use of *Dalton's law of additive pressures*, which states that the pressure of a gas mixture, P_m, is equal to the sum of the pressures each gas would exert if it existed alone at the mixture temperature and volume. This is stated mathematically in Eq1, where P_{02} and P_{N2} are the partial pressures of O_2 and N_2, respectively.

$$P_m = P_{O2} + P_{N2} \qquad \text{(Eq1)}$$

As Dalton's Law states, P_{02} and P_{N2} are the pressures each gas would exert if it existed alone at the mixture temperature and volume. We are given the mixture temperature as 25°C in the problem statement, but we must determine the total volume of the mixture. Since the gases are contained in rigid tanks, the total volume of the mixture will be equal to the volume initially occupied by the N_2 and O_2 as stated in Eq2.

$$V_m = V_{O2,1} + V_{N2,1} \qquad \text{(Eq2)}$$

We are given the initial volume of the N_2 as 2 m^3. We can calculate the initial volume of the O_2 from application of the ideal gas law. The value of R_{02} was found in Table A-2 as 0.2598 kJ/kg*K.

$$V_{O2,1} = \frac{m_{O2} R_{O2} T_{O2,1}}{P_{O2,1}} = \frac{(1[kg])\left(0.2598\left[\frac{kJ}{kg \bullet K}\right]\right)(288[K])}{300[kPa]} = 0.25 \text{ m}^3$$

Substituting this result into Eq2 we can determine the total volume of the mixture.

$$V_m = V_{O2,1} + V_{N2,1} = 0.25[m^3] + 2[m^3] = \mathbf{2.25\ m^3}$$

With knowledge of the mixture's volume and temperature we can use the ideal gas law with Dalton's law to determine the partial pressures of the O_2 and N_2 in the mixture. Recall we are looking for the pressure each gas would exert if it existed alone at the mixture temperature and volume.

$$P_{O2} = \frac{m_{O2} R_{O2} T_m}{V_m} = \frac{(1[kg])\left(0.2598\left[\frac{kJ}{kg \bullet K}\right]\right)(298[K])}{2.25[m^3]} = \mathbf{34.4\ kPa}$$

Before we can determine the partial pressure of the N_2 we must determine the mass of N_2. We can calculate the mass of the N_2 from application of the ideal gas law to the initial state of N_2. The value of R_{N2} was found in Table A-2 as 0.2968 kJ/kg*K.

$$m_{N2} = \frac{P_{N2,1}V_{N2,1}}{R_{N2}T_{N2,1}} = \frac{(500[kPa])(2[m^3])}{\left(0.2968\left[\frac{kJ}{kg \bullet K}\right]\right)(323[K])} = 10.43 \text{ kg}$$

$$P_{N2} = \frac{m_{N2}R_{N2}T_m}{V_m} = \frac{(10.43[kg])\left(0.2968\left[\frac{kJ}{kg \bullet K}\right]\right)(298[K])}{2.25[m^3]} = \mathbf{410\ [kPa]}$$

We can now use Eq1 to determine the pressure of the mixture.

$$P_m = P_{O2} + P_{N2} = 34.41[kPa] + 410[kPa] = \mathbf{444.4\ kPa} \quad \textbf{Answer a)}$$

An alternative approach to a)

We could have alternatively solved for the pressure of the mixture using the ideal gas law with the mole number of the mixture. We can determine the mole number of the O_2 from the mass, m_{02}, divided by the molar mass (Table A-1 lists the molar masses from many substances), M_{02}. Similarly, we can determine the mole number for N_2.

$$N_{O2} = \frac{m_{O2}}{M_{O2}} = \frac{1[kg]}{32\left[\frac{kg}{kmol}\right]} = 0.03125 \text{ kmol}$$

$$N_{N2} = \frac{m_{N2}}{M_{N2}} = \frac{10.43[kg]}{28.013\left[\frac{kg}{kmol}\right]} = 0.372 \text{ kmol}$$

The mole number of the mixture is the sum of the mole numbers of the components gases.

$$N_m = N_{O2} + N_{N2} = 0.03125[kmol] + 0.372[kmol] = 0.4035 \text{ kmol}$$

Since a mixture of two ideal gases can be, itself, treated as an ideal gas we can use the ideal gas law to determine the pressure of the mixture.

$$P_m = \frac{N_m R_u T_m}{V_m} = \frac{(0.4035[kmol])\left(8.314\left[\frac{kJ}{kmol \bullet K}\right]\right)(298[K])}{2.25[m^3]}$$

= **444.4 kPa** **Answer a)**

Part b)
To determine the heat transfer from state 1 to 2 we can write an energy balance for the system.

$$E_{m,1} - Q = E_{m,2} \rightarrow Q = (U_{m,1} + KE_{m,1} + PE_{m,1}) - (U_{m,2} + KE_{m,2} + PE_{m,2}) \rightarrow Q$$

With our assumption that ΔKE and ΔPE are approximately zero, the heat transfer can be determined from Eq3. Note: The internal energy of the mixture is the sum of the internal energies of the component gases.

$$Q = U_{m,1} - U_{m,2} = (U_{O2,1} - U_{O2,2}) + (U_{N2,1} - U_{N2,2}) \qquad \text{(Eq3)}$$

Since we have modeled the gases as ideal, the specific internal energies will be a function of temperature only → u_1-u_2 = $c_v(T_1 - T_2)$. The values of $c_{v,O2}$ and $c_{v,N2}$ were found in Table A-5 as 0.662 kJ/kg*K and 0.745 kJ/kg*K, respectively.

mpournaz 11-1-6 10:41 AM
Deleted: 2

mpournaz 11-1-6 10:41 AM
Deleted: 658

mpournaz 11-1-6 10:41 AM
Deleted: 743

O_2

$$U_{O2,1} - U_{O2,2} = m_{O2}(u_{O2,1} - u_{O2,2}) = m_{O2}c_{v,O2}(T_{O2,1} - T_{O2,2}) = (1[kg])(0.662\left[\frac{kJ}{kg \bullet K}\right](288[K] - 298[K]) = -6.62 \text{ kJ}$$

mpournaz 11-1-6 10:41 AM
Deleted: 58

N_2

$$U_{N2,1} - U_{N2,2} = m_{N2}(u_{N2,1} - u_{N2,2}) = m_{N2}c_{v,N2}(T_{N2,1} - T_{N2,2}) = (10.43[kg])(0.745\left[\frac{kJ}{kg \bullet K}\right](323[K] - 298[K]) = 194.25 \text{ kJ}$$

mpournaz 11-1-6 10:41 AM
Deleted: 193.74

Substituting these results in to Eq3, we can calculate the heat transfer from the system.

Q = -6.62 [kJ] + 194.25 [kJ] = **187.63 kJ** **Answer b)**

mpournaz 11-1-6 10:42 AM
Deleted: 58

mpournaz 11-1-6 10:42 AM
Deleted: 193.74

mpournaz 11-1-6 10:42 AM
Deleted: 2

Part c)

To determine the entropy generation from state 1 to 2 we can write an entropy balance for the system as shown in Eq4.

$$S_{m,in} - S_{m,out} + S_{gen,m} = \Delta S_{system,m} \qquad \text{(Eq4)}$$

Since there is heat transfer out of the system $S_{m,out} = Q/T_0$, ($S_{m,in} = 0$). Eq4 can be rewritten as shown in Eq5. Note: The entropy of the mixture is the sum of the entropies of the component gases.

$$S_{gen,m} = (S_{m,2} - S_{m,1}) + \frac{Q}{T_0} = \left((S_{O2,2} - S_{O2,1}) + (S_{N2,2} - S_{N2,1})\right) + \frac{Q}{T_0} \qquad \text{(Eq5)}$$

Since we have modeled the gases as ideal, we can use the ideal gas relation for change in entropy → $s_{i,2}-s_{i,1} = c_{p,i}\ln(T_{i,2}/T_{i,1})-R_i\ln(P_{i,2}/P_{i,1})$.

i = O$_2$

$$S_{O2,1} - S_{O2,2} = m_{O2}(s_{O2,1} - s_{O2,2}) = m_{O2}\left(c_{p,O2}\ln\left(\frac{T_{O2,2}}{T_{O2,1}}\right) - R_{O2}\ln\left(\frac{P_{O2,2}}{P_{O2,1}}\right)\right)$$

$$= 1[kg]\left((0.922\left[\frac{kJ}{kg \bullet K}\right]\ln\left(298[K]/288[K]\right) - \left(0.2598\left[\frac{kJ}{kg \bullet K}\right]\right)\ln\left(34.41[kPa]/300[kPa]\right)\right)$$

$$= 1[kg]\left(\left(0.0313\left[\frac{kJ}{kg \bullet K}\right]\right) - \left(-0.5626\left[\frac{kJ}{kg \bullet K}\right]\right)\right)$$ = *0.594 kJ/K* *(corrected)*

i = N$_2$

$$S_{N2,1} - S_{N2,2} = m_{N2}(s_{N2,1} - s_{N2,2}) = m_{N2}\left(c_{p,N2}\ln\left(\frac{T_{N2,2}}{T_{N2,1}}\right) - R_{N2}\ln\left(\frac{P_{N2,2}}{P_{N2,1}}\right)\right)$$

$$= 10.43[kg]\left((1.042\left[\frac{kJ}{kg \bullet K}\right]\ln\left(298[K]/323[K]\right) - \left(0.2968\left[\frac{kJ}{kg \bullet K}\right]\right)\ln\left(410[kPa]/500[kPa]\right)\right)$$

$$10.43[kg]\left(-0.0837\left(\left[\frac{kJ}{kg \bullet K}\right]\right) - \left(-.0373\left[\frac{kJ}{kg \bullet K}\right]\right)\right)$$ = *-0.260 kJ/K* *(corrected)*

mpournaz 11-1-6 10:43 AM
Deleted: *259*

Using these results with Eq5 we can determine the entropy generated.

$$S_{gen} = \left(\left(0.594\left[\frac{kJ}{K}\right]\right) + \left(-0.259\left[\frac{kJ}{K}\right]\right)\right) + \frac{187.2[kJ]}{298[K]}$$ = **0.963 kJ/k** **Answer c)**

Part d)

We can determine the exergy destroyed using the relation given in Eq6.

$$X_{destroyed} = T_0 S_{gen} \qquad \text{(Eq6)}$$

We are given the temperature of the surroundings in the problem statement and we determined the entropy generated in part c).

$$X_{destroyed} = T_0 S_{gen} = (298[K])\left(0.963\left[\frac{kJ}{K}\right]\right) = \mathbf{287\ kJ} \quad \textbf{Answer d)}$$

Step 5: Concluding Remarks & Discussion

a) the final pressure in the tank is 441.4 kPa,
b) the heat transfer out of the system is 187.2 kJ,
c) the entropy generated during this process is 0.963 kJ/K, and
d) the exergy destruction associated with this process is 287 kJ.