Problem Set for Bessel Equations and Bessel Functions **Due Date: March 11, 2004**

1. By means of power series, asymptotic expansions, polynomial approximations or Tables, compute to 6 decimal places the following Bessel functions:

a)	J_0(x)	e)	$Y_0(x)$
b)	J_1(y)	f)	$Y_1(y)$
c)	I_0(z)	g)	$K_0(z)$
d)	I_1(z)	h)	$K_1(z)$

given

$$\begin{aligned} x &= 3.83171 \\ y &= 2.40482 \\ z &= 1.75755 \end{aligned}$$

2. Compute to 6 decimal places the first six roots x_n of the transcendental equation

$$x\ J_1(x) - B\ J_0(x) = 0 \qquad\qquad B \geq 0$$

when $B = 0.1, \quad 1.0, \quad 10,$ and 100.

3. Compute to 4 decimal places the coefficients $A_n (n = 1,\ 2,\ 3,\ 4,\ 5,\ 6)$ given

$$A_n = \frac{2B}{(x_n^2 + B^2)\ J_0(x_n)}$$

for $B = 0.1,\ 1.0,\ 10,$ and 100. The x_n are the roots found in Problem 2.

4. Compute to 4 decimal places the coefficients $B_n (n = 1,\ 2,\ 3,\ 4)$ given

$$B_n = \frac{2 A_n J_1(x_n)}{x_n}$$

for $B = 0.1,\ 1.0,\ 10,$ and 100. The x_n are the roots found in Problem 2 and the A_n are the coefficients found in Problem 3.

5. The fin efficiency of a longitudinal fin of convex parabolic profile is given as

$$\eta = \frac{1}{\gamma} \frac{I_{2/3}(4\sqrt{\gamma}/3)}{I_{-2/3}(4\sqrt{\gamma}/3)}$$

Compute η for $\gamma = 3.178$.

6. The fin efficiency of a longitudinal fin of triangular profile is given by

$$\eta = \frac{1}{\gamma} \frac{I_1(2\sqrt{\gamma})}{I_0(2\sqrt{\gamma})}$$

Compute η for $\gamma = 0.5,\ 1.0,\ 2.0,\ 3.0,$ and 4.0.

7. The fin efficiency of a radial fin of rectangular profile is given by

$$\eta = \frac{2\rho}{x(1-\rho^2)} \left\{ \frac{I_1(x)K_1(\rho x) - K_1(x)I_1(\rho x)}{I_0(\rho x)K_1(x) - I_1(x)K_0(\rho x)} \right\}$$

Compute the efficiency for $x = 2.24, \quad \rho x = 0.894$.

Ans: $\eta = 0.537$

8. Show that

i) $J_0'(x) = J_1(x)$

ii) $\dfrac{d}{dx}[xJ_1(x)] = xJ_0(x)$

9. Given the function

$$f(x) = xJ_1(x) - B\ J_0(x) \qquad \text{with} \quad B = \text{constant} \geq 0$$

determine $f',\ f'',$ and f'''. Reduce all expressions to functions of $J_0(x),\ J_1(x)$ and B only.

10. Show that

i) $\displaystyle\int x^2 J_0(x)\ dx = x^2 J_1(x) + xJ_0(x) - \int J_0(x)\ dx$

ii) $\displaystyle\int x^3 J_0(x)\ dx = x(x^2-4)\ J_1(x) + 2x^2 J_0(x)$

11. Show that

i) $$\int_0^1 x J_0(\lambda x)\, dx = \frac{1}{\lambda} J_1(\lambda)$$

ii) $$\int_0^1 x^3 J_0(\lambda x)\, dx = \frac{\lambda^2 - 4}{\lambda^3}\, J_1(\lambda) + \frac{2}{\lambda^2}\, J_0(\lambda)$$

12. If δ is any root of the equation $J_0(x) = 0$, show that

i) $$\int_0^1 J_1(\delta x)\, dx = \frac{1}{\delta}$$

ii) $$\int_0^\delta J_1(x)\, dx = 1$$

13. If $\delta (> 0)$ is a root of the equation $J_1(x) = 0$ show that

$$\int_0^1 x J_0(\delta x)\, dx = 0$$

14. Given the Fourier-Bessel expansion of $f(x)$ of zero order over the interval $0 \le x \le 1$

$$f(x) = A_1\ J_0(\delta_1 x) + A_2\ J_0(\delta_2 x) + A_3\ J_0(\delta_3 x) + \ldots$$

where δ_n are the roots of the equation $J_0(x) = 0$. Determine the coefficients A_n when $f(x) = 1 - x^2$.

15. Show that over the interval $0 \le x \le 1$

$$x = 2 \sum_{n=1}^{\infty} \frac{J_1(\delta_n)}{\delta_n\ J_2(\delta_n)}$$

where δ_n are the positive roots of $J_1(x) = 0$.

16. Obtain the solution to the following second order ordinary differential equations:

i) $y'' + xy = 0$

ii) $y'' + 4x^2 y = 0$

iii) $y'' + e^{2x} y = 0$ Hint: let $u = e^x$

iv) $xy'' + y' + k^2 y = 0$ $k > 0$

v) $x^2 y'' + x^2 y' + \frac{1}{4} y = 0$

vi) $y'' + \frac{1}{x} y' - \left(1 + \frac{4}{x^2}\right) y = 0$

vii) $xy'' + 2y' + xy = 0$

17. Obtain the solution for the following problem:

$$xy'' + y' - m^2 b y = 0 \qquad 0 \le x \le b, \quad m > 0$$

with

$$y(0) \ne \infty \qquad \text{and} \qquad y(b) = y_0$$

18. Obtain the solution for the following problem:

$$x^2 y'' + 2xy' - m^2 xy = 0 \qquad 0 \le x \le b, \quad m > 0$$

with

$$y(0) \ne \infty \qquad \text{and} \qquad y(b) = y_0$$

19. Show that

$$I_{3/2}(x) = \left(\frac{2}{\pi x}\right)^{1/2} \left(\cosh x - \frac{\sinh x}{x}\right)$$

$$I_{-3/2}(x) = \left(\frac{2}{\pi x}\right)^{1/2} \left(\sinh x - \frac{\cosh x}{x}\right)$$