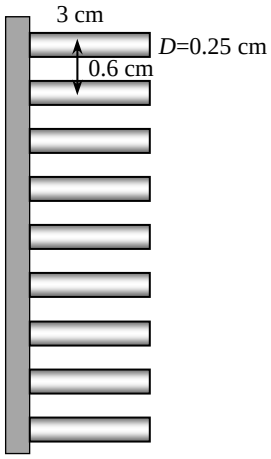
10-123 A hot plate is to be cooled by attaching aluminum pin fins on one side. The rate of heat transfer from the 1 m by 1 m section of the plate and the effectiveness of the fins are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** Heat transfer from the fin tips is negligible. **4** The heat transfer coefficient is constant and uniform over the entire fin surface. **5** The thermal properties of the fins are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the aluminum plate and fins is given to be k = 237 W/m·°C.

Analysis Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be



$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(35\ \text{W/m}^2\cdot\text{°C})}{(237\ \text{W/m.°C})(0.0025\ \text{m})}} = 15.37\ \text{m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh mL}{mL} = \frac{\tanh(15.37\ \text{m}^{-1} \times 0.03\ \text{m})}{15.37\ \text{m}^{-1} \times 0.03\ \text{m}} = 0.935$$

The number of fins, finned and unfinned surface areas, and heat transfer rates from those areas are

$$n = \frac{1\ \text{m}^2}{(0.006\ \text{m})(0.006\ \text{m})} = 27{,}777$$

$$A_{\text{fin}} = 27777\left[\pi DL + \frac{\pi D^2}{4}\right] = 27777\left[\pi(0.0025)(0.03) + \frac{\pi(0.0025)^2}{4}\right]$$
$$= 6.68\ \text{m}^2$$

$$A_{\text{unfinned}} = 1 - 27777\left(\frac{\pi D^2}{4}\right) = 1 - 27777\left[\frac{\pi(0.0025)^2}{4}\right] = 0.86\ \text{m}^2$$

$$\dot{Q}_{\text{finned}} = \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$
$$= 0.935(35\ \text{W/m}^2\cdot\text{°C})(6.68\ \text{m}^2)(100-30)\text{°C}$$
$$= 15{,}300\ \text{W}$$

$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_b - T_\infty) = (35\ \text{W/m}^2\cdot\text{°C})(0.86\ \text{m}^2)(100-30)\text{°C}$$
$$= 2107\ \text{W}$$

Then the total heat transfer from the finned plate becomes

$$\dot{Q}_{\text{total,fin}} = \dot{Q}_{\text{finned}} + \dot{Q}_{\text{unfinned}} = 15{,}300 + 2107 = 1.74 \times 10^4\ \text{W} = \mathbf{17.4\ kW}$$

The rate of heat transfer if there were no fin attached to the plate would be

$$A_{\text{no fin}} = (1\ \text{m})(1\ \text{m}) = 1\ \text{m}^2$$
$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (35\ \text{W/m}^2\cdot\text{°C})(1\ \text{m}^2)(100-30)\text{°C} = 2450\ \text{W}$$

Then the fin effectiveness becomes

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{17{,}400}{2450} = \mathbf{7.10}$$

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