10-75 An electric hot water tank is made of two concentric cylindrical metal sheets with foam insulation in between. The fraction of the hot water cost that is due to the heat loss from the tank and the payback period of the do-it-yourself insulation kit are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal resistances of the water tank and the outer thin sheet metal shell are negligible. **5** Heat loss from the top and bottom surfaces is negligible.

Properties The thermal conductivities are given to be k = 0.03 W/m·°C for foam insulation and k = 0.035 W/m·°C for fiber glass insulation

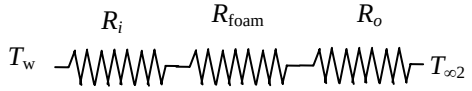
Analysis We consider only the side surfaces of the water heater for simplicity, and disregard the top and bottom surfaces (it will make difference of about 10 percent). The individual thermal resistances are

$$A_i = \pi D_i L = \pi(0.40\text{ m})(2\text{ m}) = 2.51\text{ m}^2$$

$$R_o = \frac{1}{h_i A_i} = \frac{1}{(50\text{ W/m}^2.^\circ\text{C})(2.51\text{ m}^2)} = 0.008\,^\circ\text{C/W}$$

$$A_o = \pi D_o L = \pi(0.46\text{ m})(2\text{ m}) = 2.89\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(12\text{ W/m}^2.^\circ\text{C})(2.89\text{ m}^2)} = 0.029\,^\circ\text{C/W}$$



$$R_{foam} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(23/20)}{2\pi(0.03\text{ W/m}^2\cdot^\circ\text{C})(2\text{ m})} = 0.37\,^\circ\text{C/W}$$

$$R_{total} = R_i + R_o + R_{foam} = 0.008 + 0.029 + 0.37 = 0.407\,^\circ\text{C/W}$$

The rate of heat loss from the hot water tank is

$$\dot{Q} = \frac{T_w - T_{\infty 2}}{R_{total}} = \frac{(55-27)^\circ\text{C}}{0.407\,^\circ\text{C/W}} = 68.8\text{ W}$$

The amount and cost of heat loss per year are

$$Q = \dot{Q}\Delta t = (0.0688\text{ kW})(365\times 24\text{ h/yr}) = 602.7\text{ kWh/yr}$$

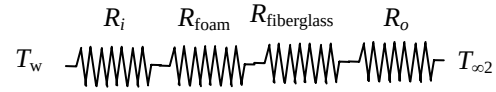
$$\text{Cost of Energy} = (\text{Amount of energy})(\text{Unit cost}) = (602.7\text{ kWh})(\$0.08/\text{kWh}) = \$48.22$$

$$f = \frac{\$48.22}{\$280} = 0.1722 = \mathbf{17.2\%}$$

If 3 cm thick fiber glass insulation is used to wrap the entire tank, the individual resistances becomes

$$A_o = \pi D_o L = \pi(0.52\text{ m})(2\text{ m}) = 3.267\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(12\text{ W/m}^2.^\circ\text{C})(3.267\text{ m}^2)} = 0.026\,^\circ\text{C/W}$$



$$R_{foam} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(23/20)}{2\pi(0.03\text{ W/m}^2\cdot^\circ\text{C})(2\text{ m})} = 0.371\,^\circ\text{C/W}$$

$$R_{fiberglass} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(26/23)}{2\pi(0.035\text{ W/m}^2\cdot^\circ\text{C})(2\text{ m})} = 0.279\,^\circ\text{C/W}$$

$$R_{total} = R_i + R_o + R_{foam} + R_{fiberglass} = 0.008 + 0.026 + 0.371 + 0.279 = 0.684\,^\circ\text{C/W}$$

The rate of heat loss from the hot water heater in this case is

$$\dot{Q} = \frac{T_w - T_{\infty 2}}{R_{total}} = \frac{(55-27)^\circ\text{C}}{0.684\,^\circ\text{C/W}} = 40.94\text{ W}$$

The energy saving is

saving = 70 - 40.94 = 29.06 W

The time necessary for this additional insulation to pay for its cost of $30 is then determined to be

Cost = (0.02906 kW)(Time period)($0.08 / kWh) = $30

Then, Time period = 12,904 hours = **538 days ≈ 1.5 years**

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