11-20 The heating times of a sphere, a cube, and a rectangular prism with similar dimensions are to be determined.

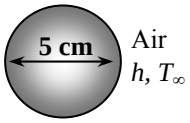
Assumptions **1** The thermal properties of the geometries are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of silver are given to be k = 429 W/m·ºC, ρ = 10,500 kg/m^3, and c_p = 0.235 kJ/kg·ºC.

Analysis For sphere, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.05\text{ m}}{6} = 0.008333\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.^\circ\text{C})(0.008333\text{ m})}{(429\text{ W/m.}^\circ\text{C})} = 0.00023 < 0.1$$



Since $Bi < 0.1$, the lumped system analysis is applicable. Then the time period for the sphere temperature to reach to 25ºC is determined from

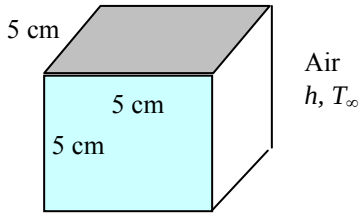
$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{12\text{ W/m}^2.^\circ\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg.}^\circ\text{C})(0.008333\text{ m})} = 0.0005836\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005836\text{ s}^{-1})t} \longrightarrow t = 2428\text{ s} = \mathbf{40.5\text{ min}}$$

Cube:

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{L^3}{6L^2} = \frac{L}{6} = \frac{0.05\text{ m}}{6} = 0.008333\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.^\circ\text{C})(0.008333\text{ m})}{(429\text{ W/m.}^\circ\text{C})} = 0.00023 < 0.1$$



$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{12\text{ W/m}^2.^\circ\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg.}^\circ\text{C})(0.008333\text{ m})} = 0.0005836\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005836\text{ s}^{-1})t} \longrightarrow t = 2428\text{ s} = \mathbf{40.5\text{ min}}$$

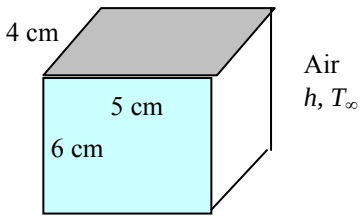
Rectangular prism:

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{(0.04\text{ m})(0.05\text{ m})(0.06\text{ m})}{2(0.04\text{ m})(0.05\text{ m}) + 2(0.04\text{ m})(0.06\text{ m}) + 2(0.05\text{ m})(0.06\text{ m})} = 0.008108\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.^\circ\text{C})(0.008108\text{ m})}{(429\text{ W/m.}^\circ\text{C})} = 0.00023 < 0.1$$

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

$$= \frac{12\text{ W/m}^2.^\circ\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg.}^\circ\text{C})(0.008108\text{ m})} = 0.0005998\text{ s}^{-1}$$



$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005998\text{ s}^{-1})t} \longrightarrow t = 2363\text{ s} = \mathbf{39.4\text{ min}}$$

The heating times are same for the sphere and cube while it is smaller in rectangular prism.

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