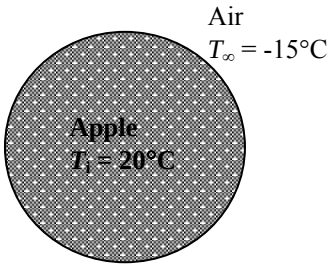
11-53 A person puts apples into the freezer to cool them quickly. The center and surface temperatures of the apples, and the amount of heat transfer from each apple in 1 h are to be determined.

Assumptions **1** The apples are spherical in shape with a diameter of 9 cm. **2** Heat conduction in the apples is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the apples are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the apples are given to be $k = 0.418$ W/m.°C, $\rho = 840$ kg/m^3, $c_p = 3.81$ kJ/kg.°C, and $\alpha = 1.3\times10^{-7}$ m^2/s.

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(8\ \text{W/m}^2.^\circ\text{C})(0.045\ \text{m})}{(0.418\ \text{W/m.}^\circ\text{C})} = 0.861$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.476 \quad \text{and} \quad A_1 = 1.2390$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.3\times10^{-7}\ \text{m}^2/\text{s})(1\,\text{h}\times3600\ \text{s/h})}{(0.045\ \text{m})^2} = 0.231 > 0.2$$

Then the temperature at the center of the apples becomes

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{T_0 - (-15)}{20 - (-15)} = (1.239)e^{-(1.476)^2(0.231)} = 0.749 \longrightarrow T_0 = \mathbf{11.2^\circ C}$$

The temperature at the surface of the apples is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\,\frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.239)e^{-(1.476)^2(0.231)}\,\frac{\sin(1.476\ \text{rad})}{1.476} = 0.505$$

$$\frac{T(r_o,t) - (-15)}{20 - (-15)} = 0.505 \longrightarrow T(r_o,t) = \mathbf{2.7^\circ C}$$

The maximum possible heat transfer is

$$m = \rho V = \rho\frac{4}{3}\pi r_o^3 = (840\ \text{kg/m}^3)\left[\frac{4}{3}\pi(0.045\ \text{m})^{3.}\right] = 0.3206\ \text{kg}$$

$$Q_{\max} = mc_p(T_i - T_\infty) = (0.3206\ \text{kg})(3.81\ \text{kJ/kg.}^\circ\text{C})\left[20 - (-15)\right]^\circ\text{C} = 42.75\ \text{kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\,\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.749)\,\frac{\sin(1.476\ \text{rad}) - (1.476)\cos(1.476\ \text{rad})}{(1.476)^3} = 0.402$$

$$Q = 0.402Q_{\max} = (0.402)(42.75\ \text{kJ}) = \mathbf{17.2\ kJ}$$

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