14-48 A vertically oriented cylindrical hot water tank is located in a bathroom. The rate of heat loss from the tank by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the outer surface of the tank is constant.

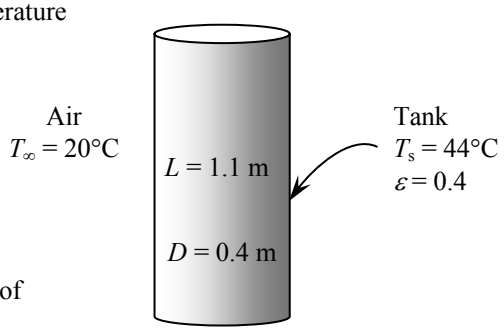
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (44+20)/2 = 32°C$ are (Table A-22)

$$k = 0.02603 \text{ W/m.°C}$$

$$\nu = 1.627\times10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.7276$$

$$\beta = \frac{1}{T_f} = \frac{1}{(32+273)\text{K}} = 0.003279 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the height of the cylinder, $L_c = L = 1.1$ m. Then,

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81 \text{ m/s}^2)(0.003279 \text{ K}^{-1})(44-20 \text{ K})(1.1 \text{ m})^3}{(1.627\times10^{-5} \text{ m}^2/\text{s})^2} = 3.883\times10^9$$

A vertical cylinder can be treated as a vertical plate when

$$D(=0.4 \text{ m}) \ge \frac{35L}{\text{Gr}^{1/4}} = \frac{35(1.1 \text{ m})}{(3.883\times10^9)^{1/4}} = 0.1542 \text{ m}$$

which is satisfied. That is, the Nusselt number relation for a vertical plate can be used for the side surfaces. For the top and bottom surfaces we use the relevant Nusselt number relations. First, for the side surfaces,

$$\text{Ra} = \text{GrPr} = (3.883\times10^9)(0.7276) = 2.825\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.825\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7276}\right)^{9/16}\right]^{8/27}}\right\}^2 = 170.2$$

$$h = \frac{k}{L}Nu = \frac{0.02603 \text{ W/m.°C}}{1.1 \text{ m}}(170.2) = 4.027 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.4 \text{ m})(1.1 \text{ m}) = 1.382 \text{ m}^2$$

$$\dot{Q}_{\text{side}} = hA_s(T_s - T_\infty) = (4.027 \text{ W/m}^2\text{.°C})(1.382 \text{ m}^2)(44-20)°C = 133.6 \text{ W}$$

For the top surface,

$$L_c = \frac{A_s}{p} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} = \frac{0.4 \text{ m}}{4} = 0.1 \text{ m}$$

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003279 \text{ K}^{-1})(44-20 \text{ K})(0.1 \text{ m})^3}{(1.627\times10^{-5} \text{ m}^2/\text{s})^2}(0.7276) = 2.123\times10^6$$

$$\text{Nu} = 0.54\text{Ra}^{1/4} = 0.54(2.123\times10^6)^{1/4} = 20.61$$

$$h = \frac{k}{L_c}Nu = \frac{0.02603 \text{ W/m.°C}}{0.1 \text{ m}}(20.61) = 5.365 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi D^2/4 = \pi(0.4 \text{ m})^2/4 = 0.1257 \text{ m}^2$$

$$\dot{Q}_{\text{top}} = hA_s(T_s - T_\infty) = (5.365 \text{ W/m}^2\text{.°C})(0.1257 \text{ m}^2)(44-20)°C = 16.2 \text{ W}$$

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For the bottom surface,

$$\text{Nu} = 0.27\text{Ra}^{1/4} = 0.27(2.123\times10^{6})^{1/4} = 10.31$$

$$h = \frac{k}{L_c}Nu = \frac{0.02603\ \text{W/m.°C}}{0.1\,\text{m}}(10.31) = 2.683\ \text{W/m}^2\text{.°C}$$

$$\dot{Q}_{\text{bottom}} = hA_s(T_s - T_\infty) = (2.683\ \text{W/m}^2\text{.°C})(0.1257\ \text{m}^2)(44-20)°\text{C} = 8.1\ \text{W}$$

The total heat loss by natural convection is

$$\dot{Q}_{\text{conv}} = \dot{Q}_{\text{side}} + \dot{Q}_{\text{top}} + \dot{Q}_{\text{bottom}} = 133.6 + 16.2 + 8.1 = \mathbf{157.9\ W}$$

The radiation heat loss from the tank is

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4}) \\ &= (0.4)(1.382 + 0.1257 + 0.1257\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\text{.K}^4)\left[(44+273\,\text{K})^4 - (20+273\,\text{K})^4\right] \\ &= \mathbf{101.1\ W}\end{aligned}$$

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