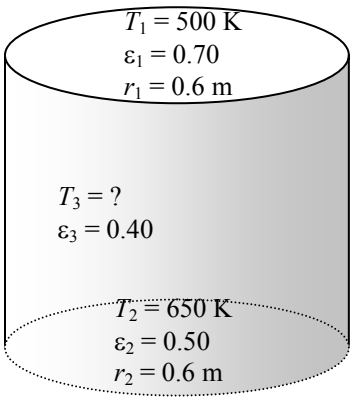
15-106 A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The temperature of the side surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of the top, bottom, and side surfaces are 0.70, 0.50, and 0.40, respectively.

Analysis We consider the top surface to be surface 1, the bottom surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from surface 1 to surface 2 is determined from

$$\left.\begin{aligned}\frac{L}{r}&=\frac{1.2}{0.6}=2\\ \frac{r}{L}&=\frac{0.6}{1.2}=0.5\end{aligned}\right\} F_{12}=0.17 \text{ (Fig. 15-7)}$$



The surface areas are

$$A_1 = A_2 = \pi D^2/4 = \pi(1.2\text{ m})^2/4 = 1.131\text{ m}^2$$

$$A_3 = \pi DL = \pi(1.2\text{ m})(1.2\text{ m}) = 4.524\text{ m}^2$$

Then other view factors are determined to be

$F_{12} = F_{21} = 0.17$

$F_{11} + F_{12} + F_{13} = 1 \longrightarrow 0 + 0.17 + F_{13} = 1 \longrightarrow F_{13} = 0.83$ (summation rule), $F_{23} = F_{13} = 0.83$

$A_1F_{13} = A_3F_{31} \longrightarrow (1.131)(0.83) = (4.524)F_{31} \longrightarrow F_{31} = 0.21$ (reciprocity rule), $F_{32} = F_{31} = 0.21$

We now apply Eq. 15-35 to each surface

Surface 1:

$$\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)(500\text{ K})^4 = J_1 + \frac{1-0.70}{0.70}\left[0.17(J_1 - J_2) + 0.83(J_1 - J_3)\right]$$

Surface 2:

$$\sigma T_2^4 = J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)(500\text{ K})^4 = J_2 + \frac{1-0.50}{0.50}\left[0.17(J_2 - J_1) + 0.83(J_2 - J_3)\right]$$

Surface 3:

$$\sigma T_3^4 = J_3 + \frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}(J_3 - J_1) + F_{32}(J_3 - J_2)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)T_3^4 = J_3 + \frac{1-0.40}{0.40}\left[0.21(J_3 - J_1) + 0.21(J_3 - J_2)\right]$$

We now apply Eq. 15-34 to surface 2

$$\dot{Q}_2 = A_2\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right] = (1.131\text{ m}^2)\left[0.17(J_2 - J_1) + 0.83(J_2 - J_3)\right]$$

Solving the above four equations, we find

$$T_3 = \mathbf{631\text{ K}},\quad J_1 = 4974\text{ W/m}^2,\quad J_2 = 8883\text{ W/m}^2,\quad J_3 = 8193\text{ W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$\dot{Q}_{21} = A_2F_{21}(J_2 - J_1) = (1.131\text{ m}^2)(0.17)(8883 - 4974)\text{W/m}^2 = \mathbf{751.6\text{ W}}$$

The rate of heat transfer between the bottom and the side surface is

$$\dot{Q}_{23} = A_2F_{23}(J_2 - J_3) = (1.131\text{ m}^2)(0.83)(8883 - 8193)\text{W/m}^2 = \mathbf{648.0\text{ W}}$$

Discussion The sum of these two heat transfer rates are 751.6 + 644 = 1395.6 W, which is practically equal to 1400 W heat supply rate from surface 2. This must be satisfied to maintain the surfaces at the specified temperatures under steady operation. Note that the difference is due to round-off error.

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