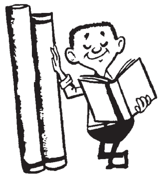
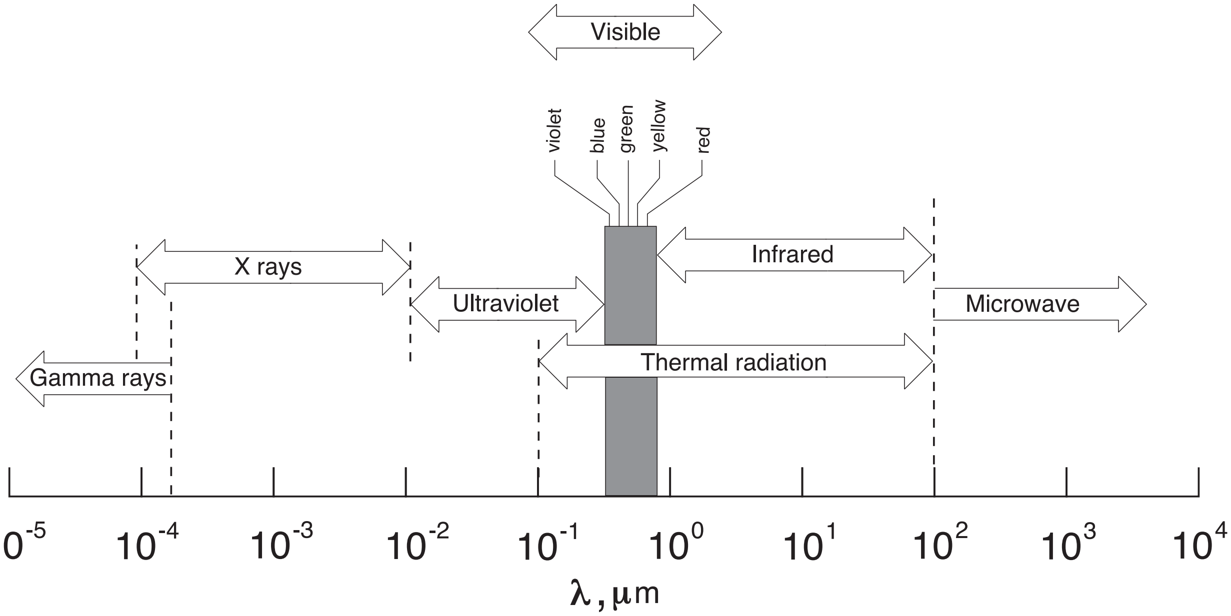
Radiation Heat Transfer



Reading	**Problems**
15-1 → 15-7	15-27, 15-33, 15-49, 15-50, 15-77, 15-79, 15-86, 15-106, 15-107

Introduction

The following figure shows the relatively narrow band occupied by thermal radiation.



An even narrower band inside the thermal radiation spectrum is denoted as the visible spectrum, that is the thermal radiation that can be seen by the human eye. The visible spectrum occupies roughly $\mathbf{0.4 - 0.7}\ \boldsymbol{\mu m}$.

Blackbody Radiation

A blackbody is an ideal radiator that

- *absorbs* all incident radiation regardless of wavelength and direction
- *emitted* radiation is a function of wavelength and temperature but is independent of direction, i.e. a black body is a <u>diffuse emitter</u> (independent of direction)

Definitions

1. **Blackbody emissive power:** the radiation emitted by a blackbody per unit time and per unit surface area

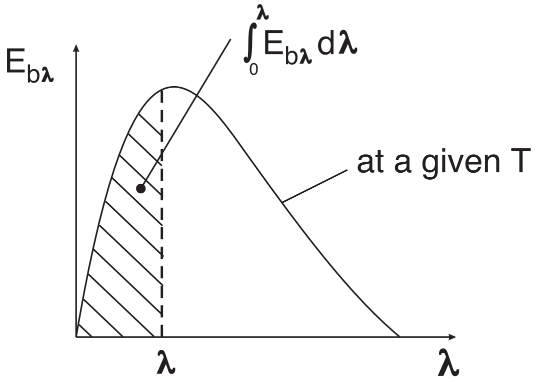
$$E_b = \sigma\, T^4 \quad [W/m^2] \qquad\qquad \Leftarrow \text{ Stefan-Boltzmann law}$$

2. **Spectral blackbody emissive power:** the amount of radiation energy emitted by a blackbody per unit surface area and per unit wavelength about the wavelength λ. The following relationship between emissive power, temperature and wavelength is known as *Plank's distribution law*

$$E_{b,\lambda} = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} \quad [W/(m^2 \cdot \mu m)]$$

3. **Blackbody radiation function:** the fraction of radiation emitted from a blackbody at temperature, T in the wavelength band $\lambda = 0 \rightarrow \lambda$

$$f_{0\rightarrow\lambda} = \frac{\displaystyle\int_0^\lambda E_{b,\lambda}(T)\, d\lambda}{\displaystyle\int_0^\infty E_{b,\lambda}(T)\, d\lambda} = \frac{\displaystyle\int_0^\lambda \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} d\lambda}{\sigma T^4}$$



let $t = \lambda T$ and $dt = T\, d\lambda$, then

$$\begin{aligned} f_{0\rightarrow\lambda} &= \frac{\displaystyle\int_0^t \frac{C_1 T^5 (1/T) dt}{t^5[\exp(C_2/t) - 1]}}{\sigma T^4} = \frac{C_1}{\sigma}\int_0^{\lambda T} \frac{dt}{t^5[\exp(C_2/t) - 1]} \\ &= f(\lambda T) \end{aligned}$$

$f_{0\rightarrow\lambda}$ is tabulated as a function λT in Table 15.2

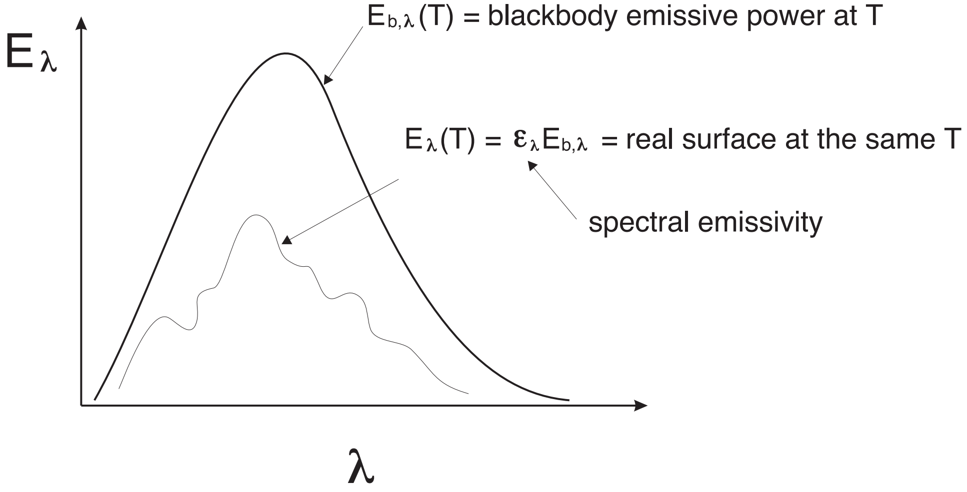
Radiation Properties of Real Surfaces

The thermal radiation emitted by a real surface is a function of surface temperature, T, wavelength, λ, direction and surface properties.

$$E_\lambda = f(T, \lambda, \textbf{direction, surface properties}) \quad \Rightarrow \text{ spectral emissive power}$$

while for a blackbody, the radiation was only a function of temperature and wavelength

$$E_{b,\lambda} = f(T, \lambda) \quad \rightarrow \quad \text{diffuse emitter} \quad \Rightarrow \text{independent of direction}$$

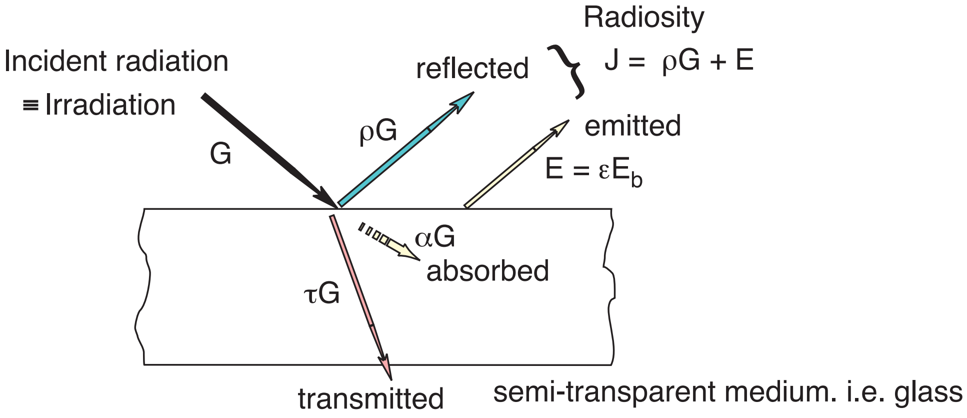


Definitions

1. **Diffuse surface:** properties are independent of direction.
2. **Gray surface:** properties are independent of wavelength.
3. **Emissivity:** defined as the ratio of radiation emitted by a surface to the radiation emitted by a blackbody at the same surface temperature.

$$\begin{aligned}\epsilon(T) &= \frac{\text{radiation emitted by surface at temperature } T}{\text{radiation emitted by a black surface at } T} \\ &= \frac{\displaystyle\int_0^\infty E_\lambda(T)\, d\lambda}{\displaystyle\int_0^\infty E_{b\lambda}(T)\, d\lambda} = \frac{\displaystyle\int_0^\infty \epsilon_\lambda(T) E_{b\lambda}(T)\, d\lambda}{E_b(T)} = \frac{E(T)}{\sigma T^4}\end{aligned}$$

4. **Irradiation, G:** the radiation energy incident on a surface per unit area and per unit time



An energy balance based on incident radiation gives

$$G = \rho G + \alpha G + \tau G$$

If we normalize with respect to the total irradiation

$$\alpha + \rho + \tau = 1$$

In general $\epsilon \neq \alpha$. However, for a diffuse-gray surface (properties are independent of wavelength and direction)

$$\epsilon = \alpha \qquad \text{diffuse-gray surface}$$

5. **Radiosity, J:** the total radiation energy leaving a surface per unit area and per unit time.

For a surface that is gray and opaque, i.e. $\epsilon = \alpha$ and $\alpha + \rho = 1$, the radiosity is given as

$$\begin{aligned} J &= \text{radiation emitted by the surface} + \text{radiation reflected by the surface} \\ &= \epsilon\, E_b + \rho G \\ &= \epsilon \sigma T^4 + \rho G \end{aligned}$$

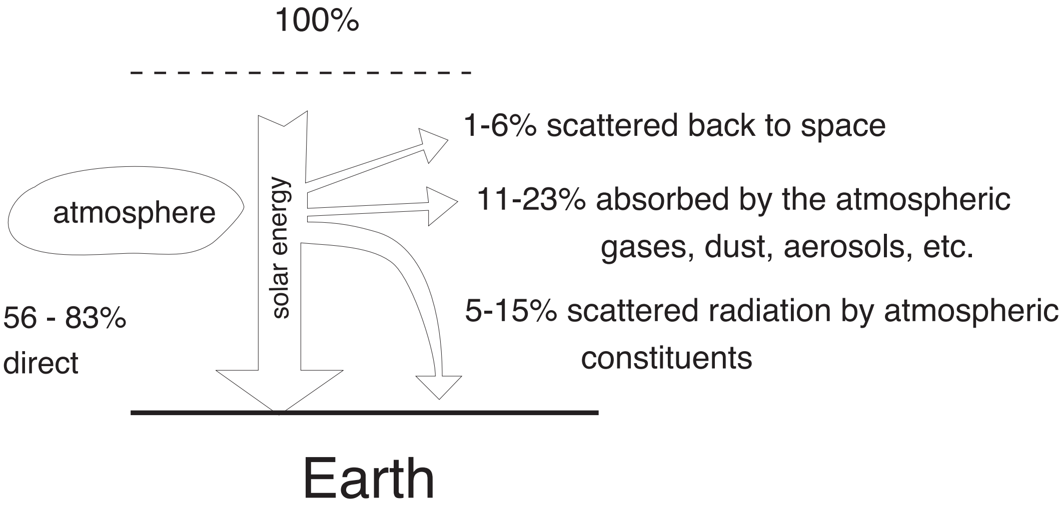
Since $\rho = 0$ for a blackbody, the radiosity of a blackbody is

$$J = \sigma T^4$$

Solar Radiation

The incident radiation energy reaching the earth's atmosphere is known as the solar constant, G_s and has a value of

$$G_s = 1353\ W/m^2$$



For most calculations we can use

$$E_{sky} \approx \sigma T_{sky}^4$$

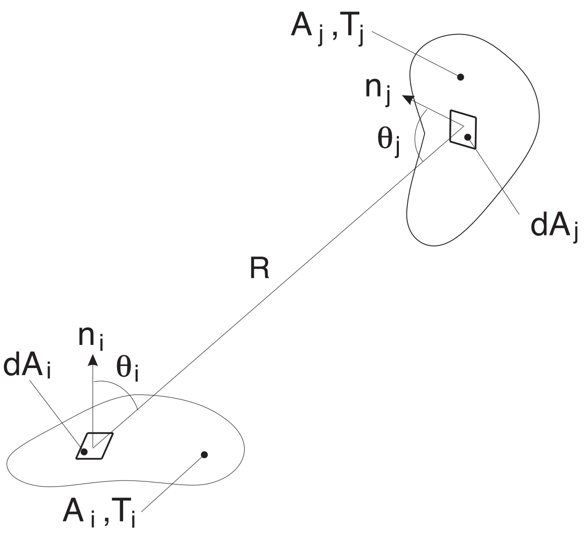
where the sky temperature is assumed to be

$$T_{sky} \approx 230\ K \text{ (cold, clear sky)} \rightarrow\ 285\ K \text{ (warm, cloudy sky)}$$

View Factor (Shape Factor, Configuration Factor)

- the radiative exchange between surfaces clearly depends on how well the surfaces "see" one another. This information is provided by using shape factors (or view factors or configuration factors).
- **Definition:** The view factor, $F_{i\rightarrow j}$ is defined as the fraction of radiation leaving surface i which is intercepted by surface j. Hence

$$F_{i\rightarrow j} = \frac{\dot{Q}_{i\rightarrow j}}{A_i J_i} = \frac{\text{radiation reaching } j}{\text{radiation leaving } i}$$



Reciprocity relation: $A_i F_{i\to j} = A_j F_{j\to i}$

Summation rule: $\sum_{j=1}^{N} F_{i\to j} = 1 \quad ; \; i = 1, 2, \ldots, N$

Note that $F_{i\to i} \neq 0$ for a concave surface. For a plane or convex surface $F_{i\to i} = 0$.

Superposition rule: if the surface is not available in the tables sometimes it can be treated as the sum of smaller known surfaces to form the full extent of the surface of interest.

$$F_{1\to(2,3)} = F_{1\to 2} + F_{1\to 3}$$

Symmetry rule: if the problem is symmetric, then the view factors will also be symmetric.

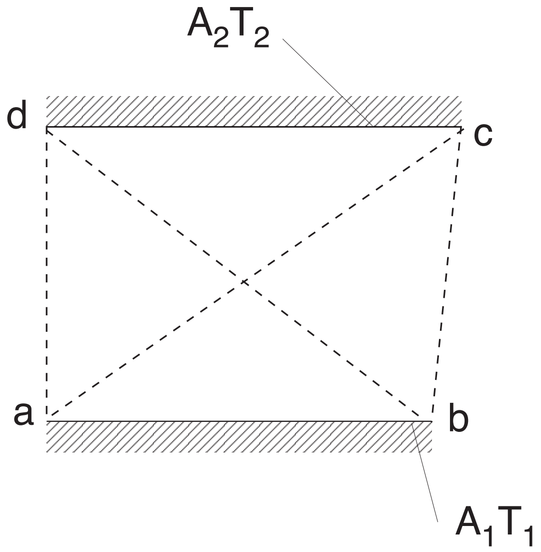
Hottel Crossed String Method

Can be applied to 2D problems where surfaces are any shape, flat, concave or convex. Note for a 2D surface the area, A is given as a length times a unit width.

$$A_1 F_{12} = A_2 F_{21} = \frac{\text{(total crossed)} - \text{(total uncrossed)}}{2}$$

A_1 and A_2 <u>do not have to be parallel</u>

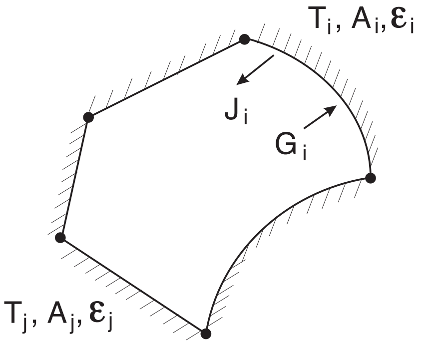
$$A_1 F_{12} = A_2 F_{21} = \frac{1}{2}[\underbrace{(ac + bd)}_{crossed} - \underbrace{(bc + ad)}_{uncrossed}]$$

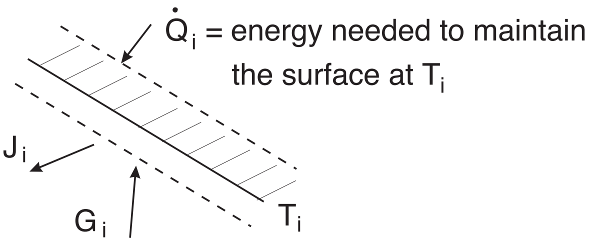


Radiation Exchange Between Diffuse-Gray Surfaces Forming an Enclosure

We will assume that:

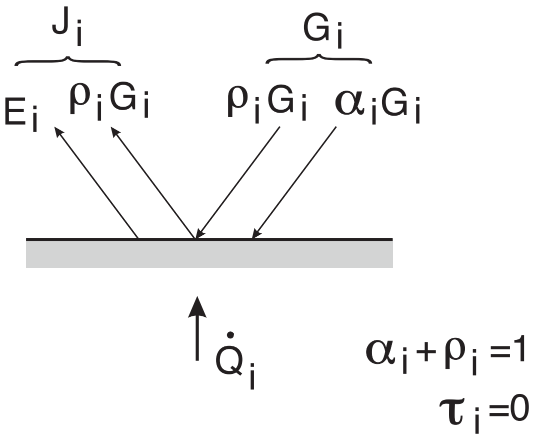
1. each surface of the enclosure is isothermal
2. radiosity, J_i, and irradiation, G_i are uniform over each surface
3. the surfaces are opaque ($\tau_i = 0$) and diffuse-gray ($\alpha_i = \epsilon_i$)
4. the cavity is filled with a fluid which does not participate in the radiative exchange process





- an energy balance on the $i'th$ surface gives:

$$\dot{Q}_i = \dot{q}_i A_i = A_i(J_i - G_i)$$



$$
\begin{aligned}
\dot{Q}_i &= A_i(E_i - \alpha_i G_i) && (1)\\
J_i &= E_i + \rho_i G_i && (2)\\
E_i &= \epsilon_i E_{b,i} = \epsilon_i \sigma T_i^4 && (3)\\
\rho_i &= 1 - \alpha_i = 1 - \epsilon_i && (4) \quad \Rightarrow \text{since } \alpha_i + \rho_i + \cancelto{0}{\tau_i} = 1\\
&&& \qquad \text{and} \quad \alpha_i = \epsilon_i
\end{aligned}
$$

Combining Eqs. 2, 3 and 4 gives

$$
J_i = \epsilon_i E_{b,i} + (1 - \epsilon_i) G_i \qquad (5)
$$

Combining this with Eq. 1 gives

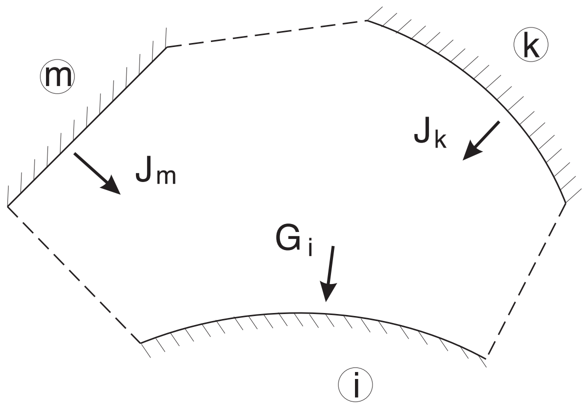
$$
Q_i = \frac{E_{b,i} - J_i}{\left(\dfrac{1 - \epsilon_i}{\epsilon_i A_i}\right)} \equiv \frac{\text{potential difference}}{\text{surface resistance}}
$$

This surface resistance represents real surface behavior as opposed to a blackbody surface. Recall that for a blackbody $\epsilon_i = \alpha_i = 1$ and $J_i = E_1 = E_{b,i} = \sigma T_i^4$.

- next consider radiative exchange between the surfaces.

  By inspection it is clearly seen that

$$
\left\{ \begin{array}{c} \text{irradiation on} \\ \text{surface } i \end{array} \right\} = \left\{ \begin{array}{c} \text{radiation leaving the} \\ \text{remaining surfaces} \end{array} \right\}
$$



$$A_i G_i = \sum_{j=1}^{N} F_{j\rightarrow i}(A_j J_j) = \sum_{j=1}^{N} A_i F_{i\rightarrow j} J_i$$

Therefore

$$G_i = \sum_{j=1}^{N} F_{i\rightarrow j} J_j$$

Combining this with Eq. 5 gives

$$J_i = \epsilon_i \sigma T_i^4 + (1-\epsilon_i)\sum_{j=1}^{N} F_{i\rightarrow j} J_j$$

In addition we can write

$$\dot{Q}_i = A_i J_i - \sum_{j=1}^{N} A_i F_{i\rightarrow j} J_j$$

Since the summation rule states $\sum_{j=1}^{N} F_{i\rightarrow j} = 1$, the above equation becomes

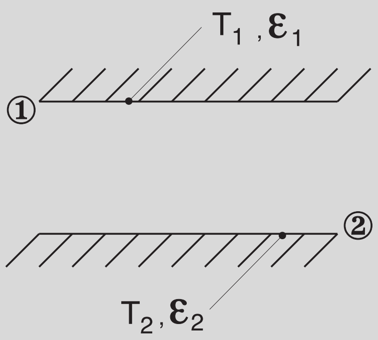
$$\dot{Q}_i = A_i \left\{ \sum_{j=1}^{N} F_{i\rightarrow j} J_i - \sum_{j=1}^{N} F_{i\rightarrow j} J_j \right\}$$

$$\dot{Q}_i = \sum_{j=1}^{N} A_i F_{i\rightarrow j}(J_i - J_j) = \sum_{j=1}^{N} \dot{Q}_{i\rightarrow j}$$

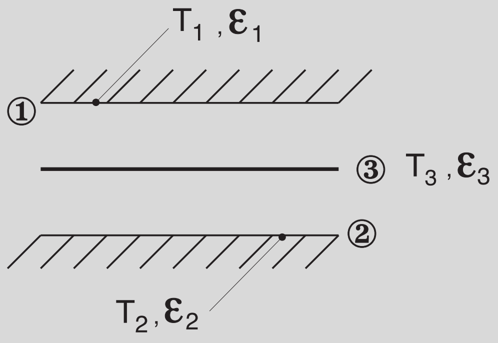
or

$$\dot{Q}_i = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(\dfrac{1}{A_i F_{i\rightarrow j}}\right)} \equiv \frac{\text{potential difference}}{\text{space resistance}}$$

Example 7-1: Consider two very large parallel plates with diffuse, gray surfaces, Determine the net rate of radiation heat transfer per unit surface area, $\dot{Q}_{12}/A$, between the two surfaces. For Case 2, also determine T_3, the temperature of a radiation shield, positioned midway between surfaces 1 and 2.



CASE 1



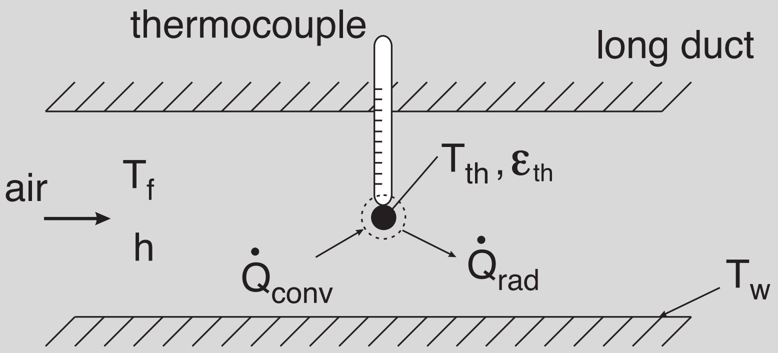
CASE 2

Given:

$$\begin{array}{lcl@{\qquad\qquad}lcl}
\epsilon_1 & = & 0.2 & T_1 & = & 800\ K \\
\epsilon_2 & = & 0.7 & T_2 & = & 500\ K \\
\epsilon_3 & = & 0.02 & A_1 & = & A_2 = A_3 = A
\end{array}$$

Assume steady state conditions.

Example 7-2: A thermocouple is suspended between two parallel surfaces as shown in the figure below. Find T_f, the temperature of the air stream by performing an energy balance on the thermocouple.



Given:

$$T_w = 400 \ K \qquad T_{th} = 650 \ K$$
$$\epsilon_{th} = 0.6 \qquad h = 80 \ W/(m^2 \cdot K)$$

Assume steady state conditions.

Example 7-3: Consider a room that is $4\ m$ long by $3\ m$ wide with a floor-to-ceiling distance of $2.5\ m$. The four walls of the room are well insulated, while the surface of the floor is maintained at a uniform temperature of $30\ °C$ using an electric resistance heater. Heat loss occurs through the ceiling, which has a surface temperature of $12°C$. All surfaces have an emissivity of 0.9.

a) determine the rate of heat loss, (W), by radiation from the room.
b) determine the temperature, (K), of the walls.