30996FE.TEX

UNIVERSITY OF WATERLOO
DEPARTMENT OF ELECTRICAL ENGINEERING
ECE 309 Thermodynamics Electrical Engineering

Final Examination Spring 1996
M.M. Yovanovich August 8, 1996 2:00 - 5:00 P.M.

NOTE:

1. Open book examination. You are permitted to use your text, your class notes and your calculator. No assignment solutions are allowed.
2. Clear systematic solutions are required. Process diagrams and sketches of equipment showing locations referred to in your analysis are both essential. **Marks will not be assigned** for problems that require unreasonable (in the opinion of your professor) effort for the marker to decipher.
3. Very briefly state the assumptions that are made in each problem.
4. Ask for clarification if any problem statement is unclear.
5. All problems are of equal value. You must answer all problems.
6. Important thermodynamic and heat transfer relationships are provided for your convenience.
7. Do not interpolate in the tables. Use the nearest points.
8. Good luck in this exam.

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1. Consider the following thermodynamic problems.

   (1a) Water is at the temperature $T = 30\ ^\circ C$ and a pressure of $P = 10\ MPa$. Find or calculate

   (a) its specific internal energy;

   (b) its specific enthalpy;

   (c) its specific entropy.

   (1b) Find the change in the specific internal energy, enthalpy and entropy of air between the initial state: $(T_1 = 100\ ^\circ C, P_1 = 0.01\ atm)$ to the final state: $(T_2 = 400\ ^\circ C, P_2 = 1.4\ atm)$. Model the air as an ideal gas.

   (1c) A mass m_1 of a liquid at absolute temperature T_1 is mixed with a mass m_2 of the same liquid at absolute temperature T_2. The system is thermally insulated to heat transfer. If $m_1 = m_2 = m$ and $c_{p,1} = c_{p,2} = c_p$ during the mixing process,

   (a) Obtain the expression for the equilibrium temperature of the system.

   (b) Show that the net entropy change during the mixing process is

$$S_2 - S_1 = 2mc_p \ln \left[\frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}} \right]$$

2. A cylinder whose volume is $2\ m^3$ contains saturated ammonia at a temperature of $40\ ^\circ C$. Initially the cylinder contains 50% liquid and 50% vapor by volume. Vapor is withdrawn from the top of the cylinder until the temperature of the mixture in the cylinder drops to $10\ ^\circ C$.

   Assuming that only vapor is removed from the cylinder and that the process is adiabatic, calculate the mass of ammonia that is removed from the cylinder.

   The ammonia tables (adapted from National Bureau of Standards Circular No. 142) provide the following properties for the two temperature levels:

$$\begin{aligned} v_{f1} &= 0.001726\ m^3/kg \\ v_{g1} &= 0.0833\ m^3/kg \\ h_{f1} &= 371.7\ kJ/kg \\ h_{g1} &= 1472.2\ kJ/kg \end{aligned}$$

$$\begin{aligned} v_{f2} &= 0.001601\ m^3/kg \\ v_{g2} &= 0.2056\ m^3/kg \\ h_{f2} &= 227.6\ kJ/kg \\ h_{g2} &= 1453.3\ kJ/kg \end{aligned}$$

The corresponding pressures are $P_1 = 1.554\ MPa$ and $P_2 = 0.61495\ MPa$

Hint: Since the saturated vapor enthalpy values at the two temperature levels differ by approximately 1.3%, use the average value during the process.

3. A simple steam power plant is shown in following Figure. It operates on 20 kg/s of steam . Neglecting losses in the various components of the system, calculate

   (a) the required pump power, $\dot{W}_P$,

   (b) the boiler heat tranfer rate, $\dot{Q}_b$,

   (c) the steam turbine power output, $\dot{W}_T$,

   (d) the condenser heat transfer rate, $\dot{Q}_c$,

   (e) the steam velocity in the boiler exit pipe,

   (f) the thermal efficiency of the cycle which is defined as
   $$\eta = (\dot{W}_T - \dot{W}_P)/\dot{Q}_b.$$

Figure: Schematic of Simple Steam Power Plant

4. For a Carnot refrigerator (a two temperature heat pump) operating between two temperatures: T_L, T_H, the coefficient of performance COP is defined as

$$COP = \frac{Q_L}{W} = \frac{1}{T_H/T_L - 1}$$

where Q_L is the heat transferred at the low temperature T_L and the high temperature is T_H.

   (a) Sketch the two temperature heat pump showing T_L, T_H, Q_L, Q_H, W. Show clearly the direction of the energy transfers.

   (b) Define a control volume and by means of the First Law of Thermodynamics **FLOT** find the relationship between Q_H and Q_L, W.

(c) By means of the Second Law of Thermodynamics **SLOT** find the relation between Q_L, Q_H, T_L, T_H.

(d) A refrigeration unit is used to cool a space to $T_L = -5°C$ by rejecting energy Q_H to the atmosphere at $T_H = 20°C$. Another refrigeration unit is required to reduce the temperature in the refrigerated space to $-25°C$.

Calculate the minimum percentage increase in the work W, for the same amount of energy removed Q_L, to accomplish this. Assume a Carnot refrigerator.

5. A rear window defroster in an automobile consists of uniformly distributed high-resistance (electrical) wires molded into the glass whose thermal conductivity k_g is assumed to be constant. When power is applied to the wires, uniform heat generation may be assumed to occur throughout the thickness t_g of the window. During operation, the heat which is generated is transferred by convection heat transfer from both the exterior and interior surfaces of the window into the atmosphere and the interior of the automobile. However, due to the vehicle speed and atmospheric winds, the heat transfer coefficient on the interior surface h_i is much smaller than the coefficient on the exterior surface h_o.

For the calculations to follow use the system parameters: $t_g = 3mm, k_g = 1.4W/m \cdot K, T_{fi} = 18°C, T_{fo} = -10°C, h_i = 6W/m^2 \cdot K, h_o = 55W/m^2 \cdot K$. Take the volumetric heat generation rate to be $5 \times 10^5 W/m^3$ and base all calculations on a unit surface area of the window. Ignore radiation heat transfer at the interior and exterior surfaces.

(a) Sketch the steady-state temperatures within the the window *before* the defroster has been turned on and *after* the defroster has been on for some time. Assume that $T_{fi} > T_{fo}$ where T_{fi} is the inner temperature and T_{fo} is the exterior temperature.

(b) Show the thermal circuit of the system showing the thermal resistances, the temperature nodes: T_g, T_{fi}, T_{fo}, and the heat flow rates $\dot{Q}_i, \dot{Q}_o$ into the interior and the exterior respectively.

(c) Calculate the thermal resistances per unit area and find the heat loss rate from the interior of the automobile into the atmosphere when the defroster is off.

(d) Estimate the glass temperature when the defroster is operating. For this calculation assume that the temperature of the glass is uniform because its thermal resistance is much smaller than the two film resistances.

Some Equations and Relationships

1. $E_1 + W_{12} + Q_{12} = E_2$
2. $e = u + \frac{1}{2}\bar{V}^2 + gz + \cdots$
3. $h \equiv u + Pv \qquad u = \frac{U}{M} \qquad v = \frac{V}{M}$
4. $v = (1-x)v_f + xv_g \qquad v_{fg} = v_g - v_f$
5. $\left(\frac{dE}{dt}\right)_{CV} = \dot{Q} + \dot{W} + \sum_{in}(e + Pv)\dot{M} - \sum_{out}(e + Pv)\dot{M}$
6. $\left(\frac{dM}{dt}\right)_{CV} = \sum_{in}\dot{M} - \sum_{out}\dot{M}$
7. $PV = MRT$
8. $c_v \equiv \left(\frac{\partial u}{\partial T}\right)_V \qquad c_p \equiv \left(\frac{\partial h}{\partial T}\right)_P \qquad c_p = c_v + R$

Ideal Gas Relations
Variable Specific Heats

$u_2 - u_1 = \int_1^2 c_v \, dT$

$h_2 - h_1 = \int_1^2 c_P \, dT$

$s_2 - s_1 = \int_1^2 c_v \, \frac{dT}{T} + R \ln \frac{v_2}{v_1}$

$s_2 - s_1 = \int_1^2 c_P \, \frac{dT}{T} + R \ln \frac{P_2}{P_1}$

Incompressible Fluid or Solid Relations
$v =$ constant and $c_P = c_v = c$.
Variable Specific Heats

$u_2 - u_1 = \int_1^2 c \, dT$

$h_2 - h_1 = \int_1^2 c \, dT + v(P_2 - P_1)$

$s_2 - s_1 = \int_1^2 c \, \frac{dT}{T}$

Control Volume Forms of the General Conservation Equations

Continuity Equation

$$\frac{\partial}{\partial t}\left(\int_{CV} \rho dV\right) = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$

First Law of Thermodynamics

$$\frac{\partial}{\partial t}\left(\int_{CV} e\rho dV\right) = \sum_{\text{in}} \int_{\mathrm{A_{in}}} (e + Pv)\, d\dot{m} - \sum_{\text{out}} \int_{\mathrm{A_{out}}} (e + Pv)\, d\dot{m} + \dot{Q}_{CV} + \dot{W}_{CV}$$

Second Law of Thermodynamics

$$\frac{\partial}{\partial t}\left(\int_{CV} s\rho dV\right) - \sum_{\text{in}} \int_{\mathrm{A_{in}}} s\; d\dot{m} + \sum_{\text{out}} \int_{\mathrm{A_{out}}} s\; d\dot{m} - \sum_{i} \left(\frac{\dot{Q}_i}{T_i}\right)_{CV} = \mathcal{P}_{\mathrm{s}} \geq 0$$

Heat Transfer Relations

Conduction, Convection and Radiation Laws

Fourier Law of Conduction
$\vec{q} = \frac{\dot{Q}}{A} = -k\ \nabla T$
Newton Law of Cooling
$q = \frac{\dot{Q}}{A} = h(T_{\text{wall}} - T_{\text{fluid}})$
Stefan-Boltzmann Law of Radiation for Black Bodies
$\dot{Q} = \sigma A_1(T_1^4 - T_2^4)$

Thermal Resistances

Thermal resistance is generally defined as $R \equiv (T_1 - T_2)/\dot{Q}$. The units are K/W.
Conduction Resistances

Plane wall: $R = \dfrac{L}{kA}$

Cylindrical shell: $R = \dfrac{\ln(b/a)}{2\pi Lk}$

Spherical shell: $R = (1/a - 1/b)/(4\pi k)$

Fins: $R = 1/(\sqrt{hPkA}\ \tanh(mL))$ and $m = \sqrt{hP/kA}$

Fluid or Film Resistance

$R = 1/(hA)$

Radiation Resistances

Grey Surface Resistance: $R = \dfrac{(1-\epsilon)}{A\epsilon}$

Spatial Resistance: $R = \dfrac{1}{A_1 F_{12}} = \dfrac{1}{A_2 F_{21}}$

Notes: Units of radiation resistances are $1/m^2$. F_{12} is the view factor between two surfaces: A_1 and A_2 is dimensionless and its range is $0 \leq F_{12} \leq 1$. The surface emissivity ϵ is a complex radiation parameter which is determined experimentally for real (grey) surfaces. It is dimensionless and its range is $0 \leq \epsilon \leq 1$. Smooth, highly polished metals such as aluminum have values as low as $\epsilon \approx 0.01 - 0.1$. Very rough, oxidized surfaces have values as high as $\epsilon \approx 0.8 - 0.95$. Black bodies are ideal bodies for which $\epsilon = 1$.

The total radiation resistance of a two surface enclosure which is bounded by two isothermal, grey surfaces is given by:

$$R_{\text{total}} = \frac{(1-\epsilon_1)}{A_1\epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{(1-\epsilon_2)}{A_2\epsilon_2}$$

The radiation heat transfer rate between the two surfaces is given by

$$\dot{Q} = \frac{(e_{b1} - e_{b2})}{R_{\text{total}}} = \frac{\sigma(T_1^4 - T_2^4)}{R_{\text{total}}}$$