UNIVERSITY OF WATERLOO
DEPARTMENT OF ELECTRICAL ENGINEERING
ECE 309 Thermodynamics and Heat Transfer

Final Examination — Spring 1997
M.M. Yovanovich — August 5, 1997 9:00 A.M.-12:00 Noon

NOTE:

1. Open book examination. You are permitted to use your text, your calculator and two crib sheets (8.5 x 11) both sides. All other material must be left outside the room.
2. Clear systematic solutions are required. Process diagrams and sketches of equipment showing locations referred to in your analysis are both essential. Marks will not be assigned for problems that require unreasonable (in the opinion of your professor) effort for the marker to decipher.
3. Very briefly state the assumptions that are made in each problem.
4. Ask for clarification if any problem statement is unclear.
5. The weight for each problem is indicated. You must answer all problems.
6. Tables and graphical data for Nitrogen are provided for your convenience.
7. Do not interpolate in the tables. Use the nearest points.
8. Good luck in this exam.

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1. (20) A dewar (a rigid, double-walled vessel for storing cryogens such as liquid Nitrogen) has a volume V which is occupied by liquid and its saturated vapor. A thick layer of insulation around the dewar limits the heat transfer rate into the dewar from the atmosphere to a low rate of $\dot{Q}_{in}$. Initially the volume of liquid Nitrogen is $V_{liq} = 0.5\ V$ and the volume of the vapor is $V_{vap} = 0.5\ V$.

   Suppose that the vent valve at the top of the dewar is left closed so that the pressure inside the dewar rises slowly . It is estimated that the walls of the dewar will rupture when the pressure inside the container reaches reaches the critical level P_c.

   How long will it take to reach this critical pressure given the following information: $V = 0.212\ m^3$, $\dot{Q}_{in} = 4.84\ W$ and $P_c = 780\ kPa$? The initial temperature of the Nitrogen is $77.35\ K$.

2. (20) Several bottles of some liquid (beer, soda, e.g.) of total mass M which are initially at room temperature T_i are placed inside an otherwise empty refrigerator. The refrigerator is run until the temperature of the liquid falls to the level $T_f < T_i$. During a cycle the refrigerator receives energy in the form of heat from the liquid, and then discharges the energy to the surrounding room at the temperature T_H.

   Considering the liquid as an incompressible substance, develop the following expression for the minimum theoretical work $W_{\min}$ required for this cooling process:

$$W_{\min} = M c_p \left[T_f - T_i - T_H \ln \left(\frac{T_f}{T_i} \right) \right]$$

3. (20) A short circuit inside an oil filled transformer can be modeled as ohmic heating $i^2 R_e$ of the oil and the air enclosed in the transformer tank. Since the tank is closed, the $i^2 R_e$ produces a pressure increase which can lead to the bursting of the tank.

   Assume that the oil is incompressible and that all of the ohmic heating is transferred to the air.

   Develop the following relationship for the pressure rise in the transformer tank:

$$\frac{\Delta P}{\Delta t} = \frac{(k-1)}{V}\, i^2 R_e$$

   where $k = c_p/c_v$ and V is the volume of the air space. The symbol Δ denotes the change in the pressure and the time between the initial and final states. Assume that the ohmic heating occurs slowly and continously during the time interval.

4. (20) A long bare copper wire of diameter D is initially in thermal equilibrium with the surrounding air temperature T_∞. Suddenly, a constant current i begins to flow through the wire and constant and uniform ohmic heating occurs within the wire. There is heat transfer from the bare surface of the wire into the air through a constant and uniform heat transfer coefficient h. The parameter $Bi = hD/k << 1$ where k is the thermal conductivity of the wire.

   (a) Obtain the symbolic expression for the steady-state temperature rise θ_{ss} in terms of some of the following system parameters: current density J, electrical resistivity ρ_e, mass density ρ, specific heat c_p, heat transfer coefficient h, and the diameter D of the copper wire.

   (b) Calculate the steady-state temperature rise for the following case: $D = 1.5\,\mathrm{mm}$, $h = 50\, W/m^2\,K$, $k = 401\, W/mK$, $\rho = 8933\, kg/m^3$, $c_p = 385\, J/kgK$, $\rho_e = 1.72 \times 10^{-8}\,\Omega\, m$, $T_0 = 280\,K$, $T_\infty = 280\,K$, and a current of $i = 22.5\,A$.

   (c) Calculate the time (in seconds) required for the temperature rise of the wire to reach a value which is 50 % of the steady-state value, i.e. $\theta(t)/\theta_{ss} = 0.5$.

5. (20) Electronic power devices are mounted to an aluminum heat sink having an exposed surface area of 0.045 m^2 and an emissivity of $\epsilon = 0.80$. When the devices dissipate a total power of 20 W and the surroundings and the air temperature are at 27^0C, the average sink temperature is 42^0C. The thermal contact resistances at the device-heat sink interfaces are reduced to negligible levels by the application of high conductivity thermal grease. Assume that the heat dissipated by the power devices enters the heat sink and then it is transferred into the air by convection and to the surroundings by radiation.

   (i) Define a control volume around the heat sink and label the heat transfer rates through its boundaries.

   (ii) Draw a simple thermal circuit labelling the thermal nodes and the thermal resistors and the heat transfer rates.

   (iii) What average temperature will the heat sink attain when the devices dissipate 30 W for the same environmental temperature? Assume that the heat transfer coefficient h is the same for the two power levels.

Some Equations and Relationships

(a) $E_1 + W_{12} + Q_{12} = E_2$

(b) $e = u + \frac{1}{2}\bar{V}^2 + gz + \cdots$

(c) $h \equiv u + Pv \qquad u = \frac{U}{M} \qquad v = \frac{V}{M}$

(d) $v = (1-x)v_f + xv_g \qquad v_{fg} = v_g - v_f$

(e) $\left(\frac{dE}{dt}\right)_{CV} = \dot{Q} + \dot{W} + \sum_{in}(e + Pv)\dot{M} - \sum_{out}(e + Pv)\dot{M}$

(f) $\left(\frac{dM}{dt}\right)_{CV} = \sum_{in}\dot{M} - \sum_{out}\dot{M}$

(g) $PV = MRT$

(h) $c_v \equiv \left(\frac{\partial u}{\partial T}\right)_V \qquad c_p \equiv \left(\frac{\partial h}{\partial T}\right)_P \qquad c_p = c_v + R$

Ideal Gas Relations
Variable Specific Heats

$u_2 - u_1 = \int_1^2 c_v \, dT$

$h_2 - h_1 = \int_1^2 c_P \, dT$

$s_2 - s_1 = \int_1^2 c_v \, \frac{dT}{T} + R \, \ln \frac{v_2}{v_1}$

$s_2 - s_1 = \int_1^2 c_P \, \frac{dT}{T} + R \, \ln \frac{P_2}{P_1}$

Incompressible Fluid or Solid Relations
v = constant and $c_P = c_v = c$.
Variable Specific Heats

$u_2 - u_1 = \int_1^2 c \, dT$

$h_2 - h_1 = \int_1^2 c \, dT + v(P_2 - P_1)$

$s_2 - s_1 = \int_1^2 c \, \frac{dT}{T}$

Control Volume Forms of the General Conservation Equations

Continuity Equation

$$\frac{\partial}{\partial t}\left(\int_{CV} \rho dV\right) = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$

First Law of Thermodynamics

$$\frac{\partial}{\partial t}\left(\int_{CV} e\rho dV\right) = \sum_{\text{in}} \int_{\mathbf{A}_{\text{in}}} (e+Pv)\ d\dot{m} - \sum_{\text{out}} \int_{\mathbf{A}_{\text{out}}} (e+Pv)\ d\dot{m} + \dot{Q}_{CV} + \dot{W}_{CV}$$

Second Law of Thermodynamics

$$\frac{\partial}{\partial t}\left(\int_{CV} s\rho dV\right) - \sum_{\text{in}} \int_{\mathbf{A}_{\text{in}}} s\ d\dot{m} + \sum_{\text{out}} \int_{\mathbf{A}_{\text{out}}} s\ d\dot{m} - \sum_{i} \left(\frac{\dot{Q}_i}{T_i}\right)_{CV} = \mathcal{P}_{\text{s}} \geq 0$$

Heat Transfer Relations

Conduction, Convection and Radiation Laws

Fourier Law of Conduction
$\vec{q} = \frac{\dot{Q}}{A} = -k\ \nabla T$

Newton Law of Cooling
$q = \frac{\dot{Q}}{A} = h(T_{\text{wall}} - T_{\text{fluid}})$

Stefan-Boltzmann Law of Radiation for Black Bodies
$\dot{Q} = \sigma A_1(T_1^4 - T_2^4)$

Thermal Resistances

Thermal resistance is generally defined as $R \equiv (T_1 - T_2)/\dot{Q}$. The units are K/W.

Conduction Resistances

Plane wall: $R = \dfrac{L}{kA}$

Cylindrical shell: $R = \dfrac{\ln(b/a)}{2\pi Lk}$

Spherical shell: $R = (1/a - 1/b)/(4\pi k)$

Fins: $R = 1/(\sqrt{hPkA}\ \tanh(mL))$ and $m = \sqrt{hP/kA}$

Fluid or Film Resistance

$R = 1/(hA)$

Radiation Resistances

Grey Surface Resistance: $R = \dfrac{(1-\epsilon)}{A\epsilon}$

Spatial Resistance: $R = \dfrac{1}{A_1 F_{12}} = \dfrac{1}{A_2 F_{21}}$

Notes: Units of radiation resistances are $1/m^2$. F_{12} is the view factor between two surfaces: A_1 and A_2 is dimensionless and its range is $0 \leq F_{12} \leq 1$. The surface emissivity ϵ is a complex radiation parameter which is determined experimentally for real (grey) surfaces. It is dimensionless and its range is $0 \leq \epsilon \leq 1$. Smooth, highly polished metals such as aluminum have values as low as $\epsilon \approx 0.01 - 0.1$. Very rough, oxidized surfaces have values as high as $\epsilon \approx 0.8 - 0.95$. Black bodies are ideal bodies for which $\epsilon = 1$.

The total radiation resistance of a two surface enclosure which is bounded by two isothermal, grey surfaces is given by:

$$R_{\text{total}} = \frac{(1-\epsilon_1)}{A_1 \epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{(1-\epsilon_2)}{A_2 \epsilon_2}$$

The radiation heat transfer rate between the two surfaces is given by

$$\dot{Q} = \frac{(e_{b1} - e_{b2})}{R_{\text{total}}} = \frac{\sigma(T_1^4 - T_2^4)}{R_{\text{total}}}$$