UNIVERSITY OF WATERLOO

DEPARTMENT OF ELECTRICAL ENGINEERING
ECE 309 Thermodynamics and Heat Transfer

Final Examination Solutions Spring 1997
M.M. Yovanovich August 5, 1997 9:00 A.M.-12:00 Noon

Problem 1

• $W = 0$, $\Delta PE = 0$, $\Delta KE = 0$
• State 1: $V_{liq} = 0.5V$, $V_{vap} = 0.5V$, $V = 0.212\,m^3$, $T_i = 77.35K$
• State 2: $P_c = 780$ $[kPa]$
• FLOT/CM gives $\dot{Q}_{in}\Delta t = \Delta U = M_{total}(u_2 - u_1)$;
$\dot{Q}_{in} = 4.84\,W$
• $M_{liq} = (0.5)(0.212)/0.001237 = 85.69\,[kg]$
• $M_{vap} = (0.5)(0.212)/0.2168 = 0.489\,[kg]$
• $M_{total} = 86.18\,[kg]$
• $x_1 = M_{vap}/M_{total} = 0.00567$
• State 1: $P_1 = 0.101325\ [MPa]\,, h_1 = 29.178\,[kJ/kg]\ h_1 = 28.93\ [kJ/kg]\,, v_1 = 0.00246\ [m^3/kg]$
• State 2: $v_2 = v_1 = 0.00246\ [m^3/kg]\,, P_2 = P_c = 780\ [kPa]$
• Find $x_2 = 0.00338$
• Find $h_2 = 82.23\ [kJ/kg]\,, u_2 = h_2 - P_2 v_2 = 80.31\ [kJ/kg]$
• $\Delta t = M(u_2 - u_1)/\dot{Q}_{in} = 914.86 \times 10^3 s$

Problem 2

• Fixed mass system: $M_{in} = M_{out} = 0$. $\Delta PE = 0$, $\Delta KE = 0$
• Consider contents and refrigerator as a heat pump operating with three temperatures: T_i, T_f, T_H. Provide simple sketch of system with control surface.
• Refrigerator is a cyclic device: $\Delta U_{refrig} = 0$
• FLOT/CV gives: $W = Q_H + Mc_p(T_f - T_i)$ where contents are incompressible.
• SLOT/CV gives: $\Delta S = -Q_H/T_H + \mathcal{P}_s$ where $\Delta S_{refrig} = 0$
• Combine results by eliminating Q_H to get $Mc_p \ln(T_f/T_i) = -(W - Mc_p(T_f - T_i)/T_H) + \mathcal{P}_s$
• $W_{min} = Mc_p\,[T_f - T_i - T_H \ln(T_f/T_i)]$ when $\mathcal{P}_s = 0$

Problem 3

• Volume is constant. Air is an ideal gas. Oil is incompressible. $W = 0$. All ohmic heating is transferred to air.
• FLOT/CM gives: $\dot{Q}\Delta t = \Delta U$
• $i^2 R_e \Delta t = M(u_2 - u_1) = M c_v (T_2 - T_1)$
• $i^2 R_e \Delta t = \frac{P_1 V}{R} c_v (T_2 - T_1)$
• $i^2 R_e \Delta t = \frac{V}{R} c_v (P_2 - P_1) = \frac{V}{R} c_v \Delta P$
• $\frac{\Delta P}{\Delta t} = i^2 R_e \frac{R}{V c_v}$
• $c_p - c_v = R$, $k = c_p / c_v$
• $\frac{\Delta P}{\Delta t} = i^2 R_e (k-1) \frac{1}{V}$

Problem 4

• (a)
• Steady-state temperature rise. $\dot{Q}_{gen} = \dot{Q}_{loss}$
• $i^2 R_e = h P L \theta_{ss}$
• $i^2 \rho_e L / A_c^2 = h P L \theta_{ss}$
• $J^2 \rho_e = h \pi D \theta_{ss}$
• $\theta_{ss} = J^2 \rho_e D / (4h)$
• (b) $\theta_{ss} = 20.9K$
• (c) Lumped capacitance solution:

$$\theta(t) = \frac{n}{m} + \left(-\frac{n}{m} + \theta_0 \right) \exp(-mt)$$

where $n/m = \theta_{ss}$ and $m = hP/(\rho c_p A_c)$
• $\theta(t) = 0.5\theta_{ss} = 10.45K$, $m = 0.03877/s$, $\theta_0 = 0K$
• Solve for t to get $t = -\ln(1/2)/m = 17.88s$

Problem 5

• (i) Simple schematic should show $\dot{Q}_{device}, \dot{Q}_{conv}, \dot{Q}_{rad}$
• (ii) The thermal circuit consists of four temperature nodes:
$T_{device}, T_{heat\ sink}, T_\infty, T_{surr}$ where $T_\infty = T_{surr}$, and three resistors: $R_{heat\ sink}, R_{conv}, R_{rad}$ where R_{conv} and R_{rad} are in parallel; both are connected to the heat sink node $T_{heat\ sink}$ and the nodes: T_∞ and T_{surr}. The heat sinks resistor is connected to the nodes: T_{device} and $T_{heat\ sink}$.
• (iii) $\dot{Q}_{devices} = \dot{Q}_{conv} + \dot{Q}_{rad}$ where $\dot{Q}_{device} = 20W$, $\dot{Q}_{conv} = hA\Delta T$ and $\dot{Q}_{rad} = h_{rad} A \Delta T$. Given $A = 0.045m^2$, $\epsilon = 0.80$, $T_\infty = T_{surr} = 27°C$ and $T_{heat\ sink} = 42°C$. $\Delta T = 15°C$.
$h_{rad} = \sigma(T_1^2 + T_2^2)(T_1 + T_2)/(A R_{rad})$ with $R_{rad} = (1-\epsilon)/(A\epsilon) + 1/(A F_{12})$ and

$F_{12} = 1$
$h_{rad} = 5.27\, W/m^2 \cdot K$
h_{conv} is unknown. Can find it value from the heat balance relation. $h_{conv1} = 29.63\, W/m^2 \cdot K$.
Use heat balance relation again to find by trial and error (iteration) that average heat sink temperature increases to $T_{heat\ sink} = 49.6°C$ when the devices dissipate $\dot{Q}_{devices} = 30\, W$ and h_{conv} remains constant.