UNIVERSITY OF WATERLOO
Department of Electrical Engineering

ECE 309 Thermodynamics and Heat Transfer for Electrical Engineering

Mid-Term Examination — Spring 1998
M.M. Yovanovich — June 10, 1998, 5:00-7:00 P.M.

NOTE:

1. Open book examination. You are permitted to use your calculator, the text book, your lecture notes, and material downloaded from the Website. One crib sheet (8.5 x 11) one side only.
2. Clear systematic solutions are required. Marks will not be assigned for problems that require unreasonable (in the opinion of your professor) effort for the marker to decipher.
3. Very briefly state the assumptions that are made in each problem.
4. Ask for clarification if any problem statement is unclear.
5. All problems are of equal weight. You must answer all *three* problems.

Problem 1. An electric current i flows along a long flat plate whose cross-section $A_c = LW$. One surface of the plate, $x = 0$, is held at temperature T_1 while the opposite surface at $x = L$ is held at temperature $T_2 < T_1$. The other two edges of the plate are insulated (adiabatic). The heat which is generated within the cross-section of the plate due to ohmic heating must flow out through the two surfaces. The thermal conductivity k and the electrical resistivity ρ_e are assumed to be constant. The steady-state temperature distribution is one-dimensional, $T(x)$, and the equation which describes the temperature field is the one-dimensional Poisson equation:

$$\frac{d^2T}{dx^2} = -\frac{\mathcal{S}}{k}, \qquad 0 < x < L$$

where $\mathcal{S}$ represents the uniform volumetric heat generation rate.

(a) What is the relation between the parameter $\mathcal{S}$ and the current density J and the electrical resistivity ρ_e?

(b) Obtain the expression for the temperature distribution.

(c) Obtain the relation fo the location $x_{\max}$ of the maximum temperature $T_{\max}$ in terms of the system parameters: $T_1, T_2, k, L, \mathcal{S}$. Verify your result for the symmetric case where $T_1 = T_2$.

(d) Obtain relations for the heat transfer rates per unit area through the surfaces at $x = 0$ and $x = L$ given:

$$\frac{\dot{Q}}{A} = k\frac{dT(0)}{dx} \qquad \text{and} \qquad \frac{\dot{Q}}{A} = -k\frac{dT(L)}{dx}$$

(e) Show that the sum of the two boundary heat transfer rates per unit area is equal to $\mathcal{S}L$.

(f) Calculate the location of the maximum temperature, the maximum temperature and the boundary heat transfer rates for the system parameters: $T_1 = 95°C, T_1 = 80°C, k = 54\ W/m \cdot K, L = 12.5\ mm, W = 100\ mm, i = 34000\ amp, \rho_e = 12 \times 10^{-8}\ \Omega \cdot m$.

Problem 2. A long bare copper wire of diameter D is initially in thermal equilibrium with the surrounding air temperature T_∞. Suddenly, a constant current i begins to flow through the wire and constant and uniform ohmic heating occurs within the wire. There is heat transfer from the bare surface of the wire into the air through a constant and uniform heat transfer coefficient h. The parameter $Bi = hD/(2k) << 1$ where k is the thermal conductivity of the wire.

(a) Obtain the symbolic expression for the steady-state temperature rise θ_{ss} in terms of some of the following system parameters: current density J, electrical resistivity ρ_e, mass density ρ, specific heat c_p, heat transfer coefficient h, and the diameter D of the copper wire.

(b) Calculate the steady-state temperature rise for the following case: $D = 1.5\,\text{mm}$, $h = 50\,W/m^2 \cdot K$, $k = 401\,W/m \cdot K$, $\rho = 8933\,kg/m^3$, $c_p = 385\,J/kg \cdot K$, $\rho_e = 1.72 \times 10^{-8}\,\Omega \cdot m$, $T_0 = 280\,K$, $T_\infty = 280\,K$, and a current of $i = 22.5\,A$.

(c) Calculate the time (in seconds) required for the temperature rise of the wire to reach a value which is 50 % of the steady-state value, i.e. $\theta(t)/\theta_{ss} = 0.5$.

Problem 3. A long brass tube of inner and outer radii r_1, r_2 respectively has the following thermophysical properties: $\rho = 8500\,kg/m^3, c_p = 0.385\,kJ/(kg \cdot K), k = 110\,W/(m \cdot K)$. A hot fluid at temperature $T_{f1} = 390\,K$ flows through the tube and there is heat transfer to the inner tube wall through a film coeffcient estimated to be $h_1 = 100\,W/(m^2 \cdot K)$. The heat is transferred through the tube wall and then dissipated into the environment which consists of the surrounding air which is at the temperature $T_\infty = 300\,K$ and to the walls, ceiling and floor of the room which is at the temperature $T_{\text{surr}} = 300\,K$. The natural convection heat transfer coeffcient is estimated to be $h_{\text{conv}} = 10\,W/(m^2 \cdot K)$, and the radiation film coeffcient is estimated to be $h_{\text{rad}} = 6.5 W/(m^2 \cdot K)$. The inner and outer tube radii are: $r_1 = 6.35\,mm$ and $r_2 = 6.60\,mm$.

(a) Sketch the equivalent thermal circuit labeling all thermal nodes and thermal resistors.

(b) Calculate all thermal resistors and the overall resistance per unit length of the tube.

(c) Calculate the heat transfer rate from the system per unit length of the tube.

(d) Calculate the heat transfer rate if the outer surface of the tube is covered with insulation whose thickness is $t = 1.68\,mm$ and its thermal conductivity is $k_{\text{insul}} = 0.06\,W/(m \cdot K)$. Assume that the convective and radiative heat transfer coefficients remain unchanged.