ECE 309 M.M. Yovanovich

Week 12

Lecture 1

• Final Exam will be open book. Discussed this with Prof. Wilson.

• Entropy and the Second Law of Thermodynamics (SLOT)
• Read Chapter 6: Sections 1 through 5.
• See ECE 309 Web site for notes on Entropy.
• $S\,[J/K]$, extensive property
• $s = S/M\ [J/(kg \cdot K]$, intensive property
• Perfect crystal (pure substance) at $0\,K$, molecules are motionless and perfectly arranged; entropy is zero, i.e., $S = 0$
• Discuss $T - s$ diagram
• For mixture:

$$s = (1-x)s_f + xs_g$$

• See example in text.
• Isolated system (system plus surroundings); (boundary closed to energy transfer in the form of heat and work):

$$(\Delta S)_{\text{isolated system}} = (\Delta S)_{\text{system}} + (\Delta S)_{\text{surroundings}} \geq 0$$

• Reversible process $\Delta S = 0$ and irreversible process $\Delta S > 0$
• Production $\mathcal{P}$

$$\begin{aligned} &\text{CE} \qquad \mathcal{P}_M = M_2 - M_1 = 0 \\ &\text{FLOT} \qquad \mathcal{P}_E = E_2 - E_1 = 0 \\ &\text{SLOT} \qquad \mathcal{P}_S = S_2 - S_1 \geq 0 \end{aligned}$$

• Chapter 7 Consequences of the Second Law

• $S = S(U, V)$ and $s = s(u, v)$
• Thermodynamic Definition of Temperature

$$T = \frac{1}{(\partial S/\partial U)_V}$$

• Thermodynamic Definition of Pressure

$$P = \left(\frac{\partial S}{\partial V}\right)_U$$

• Definitions of T and P in terms of $s = s(u, v)$

$$\frac{1}{T} = \left(\frac{\partial s}{\partial u}\right)_v, \qquad \frac{P}{T} = \left(\frac{\partial s}{\partial v}\right)_u$$

Lecture 2

• Gibbs equation for SCS based on $s = s(u, v)$

$$ds = \left(\frac{\partial s}{\partial u}\right)_v du + \left(\frac{\partial s}{\partial v}\right)_u dv$$

or

$$ds = \frac{du}{T} + \frac{P}{T}dv \qquad \text{or} \qquad T ds = du + P\, dv$$

• Macroscopic Evaluation of Entropy

$$s_2 - s_1 = \int_1^2 \frac{du}{T} + \int_1^2 \frac{P}{T}dv$$

• See example in Text for entropy change from saturated liquid to saturated vapor at constant pressure and temperature.
• Entropy change of solid or liquid. If SCS is incompressible, $dv = 0$, then

$$ds = \frac{du}{T}$$

If dv is small (liquid) or negligible (solid), then

$$ds \approx \frac{du}{T} \approx c\frac{dT}{T}$$

where c is the specific heat capacity. Assuming that $c =$ constant:

$$s_2 - s_1 \approx c \ln \frac{T_2}{T_1}$$

• Example: One kilogram of water is heated from $T_1 = 20°C + 273 = 293K$ to $T_2 = 90°C + 273 = 363K$. Calculate the change in the entropy assuming constant specific heat capacity. Compare the result against the steam table value.

• Solution:

$$s_2 - s_1 \approx c \ln \frac{T_2}{T_1} \quad \left[\frac{kJ}{kg \cdot K}\right]$$

But

$$\int_1^2 du = c_v \int_1^2 dT \quad \text{and} \quad u_2 - u_1 = c_v(T_2 - T_1)$$

Therefore

$$c_v = \frac{u_2 - u_1}{T_2 - T_1}$$

¿From Table B.1a, page 622 we find

$$\text{At} \quad T_1 = 293\,K, \quad u_2 = 83.9 \left[\frac{kJ}{kg \cdot K}\right]$$

$$\text{At} \quad T_2 = 363\,K, \quad u_1 = 376.8 \left[\frac{kJ}{kg \cdot K}\right]$$

This gives

$$c_v = \frac{376.8 - 83.9}{363 - 293} = 4.184 \left[\frac{kJ}{kg \cdot K}\right]$$

and

$$s_2 - s_1 = 4.184 \ln\left(\frac{363}{293}\right) = 0.8964 \left[\frac{kJ}{kg \cdot K}\right]$$

and a better estimate is

$$s_2 - s_1 = 4.184 \ln\left(\frac{363.2}{293.2}\right) = 0.8958 \left[\frac{kJ}{kg \cdot K}\right]$$

¿From steam tables:

$$s_2 - s_1 = s_{90^\circ C} - s_{20^\circ C} = 1.1927 - 0.2965 = 0.8962 \left[\frac{kJ}{kg \cdot K}\right]$$

The agreement is good. Water (liquid) can be approximated as an incompressible substance.

• Chapter 8:
– 8.2 The Gibbs Equation
– 8.3 Equation of State for Perfect Gas
– 8.4 Other $P - v - T$ Equations for Gases

• Entropy Change for Ideal Gas
• Gibbs Equation:

$$Tds = du + Pdv$$

For ideal gas

$$du = c_v dT \quad \text{and} \quad \frac{P}{T} = \frac{R}{v}$$

Now

$$ds = c_v \frac{dT}{T} + R\frac{dv}{v}$$

and the change in entropy is

$$s_2 - s_1 = \int_1^2 c_v \frac{dT}{T} + R \ln \frac{v_2}{v_1} \quad \text{and} \quad c_v = c_v(T)$$

Assuming that $c_v =$ constant, gives

$$s_2 - s_1 \approx c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Similarly

$$Tds = dh - vdP$$

For ideal gas

$$dh = c_p dT \quad \text{and} \quad \frac{v}{T} = \frac{R}{P}$$

Now

$$ds = c_p \frac{dT}{T} - R\frac{dP}{P}$$

and the change in entropy is

$$s_2 - s_1 = \int_1^2 c_p \frac{dT}{T} - R \ln \frac{P_2}{P_1} \quad \text{and} \quad c_p = c_p(T)$$

Assuming that $c_p =$ constant, gives

$$s_2 - s_1 \approx c_p \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Lecture 3

• 15 minutes at end of lecture to do Project 2 Part 2, and to hand in Part 1.

• Relationship between specific heat capacities

$$c_p - c_v = R$$

• Approximate Polynomial Fit to Experimental Data

$$c_p = a + bT + cT^2$$

where a, b, c are correlation coefficients fitted to data. Average deviation is $\pm 1\%$ and maximum deviation is $\pm 2\%$. See Table B.16 for values for several gases.

• Definition

$$\phi(T) = \int_{T_0}^{T} \frac{c_p(T)}{T} dT, \quad T_0 = \text{some reference temperature}$$

Entropy can now be expressed as

$$s_2 - s_1 = \phi_2 - \phi_1 - R \ln \frac{P_2}{P_1}$$

See Table B.9 for properties of air. Note that $T_0 = 0\,R$.

• Section 7.7 Two Idealized Systems
– Thermal Energy Reservoir (TER) is a reservoir for disorganized energy. It has a fixed volume $dV = 0$, and its temperature is T_{TER} for all time. The TER is a non-isolated system.
• Gibbs Equation for TER

$$T_{TER}\, dS = dU \quad \text{and} \quad PdV = 0$$

• FLOT for TER

$$dU = dQ \quad \text{and} \quad dS = \frac{dQ}{T_{TER}}$$

where Q enters the TER.
