ECE 309 M.M. Yovanovich

Week 13

Lecture 1

• Mechanical Energy Reservoir (MER) is a reservoir for organized mechanical energy. MER is open to work transfer only. Its walls are adiabatic $dQ = 0$. It is entropy free.

$$dS = 0 \quad \text{for a MER}$$

• Entropy Change for a Control Mass (CM). Isolated system. CM interacts with TER and MER, and $\mathcal{P}_S \geq 0$

$$dS_{CM} + dS_{TER} + dS_{MER} = d\mathcal{P}_S \geq 0$$

but $dS_{MER} = 0$ and $dS_{TER} = -dQ/T_{TER}$. Therefore

$$dS_{CM} - \frac{dQ}{T_{TER}} = d\mathcal{P}_S \geq 0$$

$\mathcal{P}_S$ is not a property. It depends on the process path just like Q and W.
• SLOT for CM

$$dS_{CM} = \frac{dQ}{T_{TER}} + d\mathcal{P}_S$$

where the terms represent:
– Increase in entropy storage within CM
– Entropy inflow in CM
– Entropy production

• Integrating gives the total entropy change in CM

$$(S_2 - S_1)_{\text{CM}} = \frac{Q_{1\to 2}}{T_{TER}} + (\mathcal{P}_S)_{1\to 2}$$

But

$$(\mathcal{P}_S)_{1\to 2} \geq 0$$

where > 0 is the real world, and $= 0$ when all processes within isolated system are reversible (frictionless, loss less, etc).
• $(\mathcal{P}_S)_{1\to 2} = 0$ will be used to determine:

– Maximum efficiency of a cyclic device
– Minimum work reqired to run a pump
– Maximum work that can be obtained from a given system

• For any Control Mass, Reversible, Adiabatic Process

$$dS_{CM} = 0 \quad \Delta S_{CM} = S_2 - S_1 = 0$$

Lecture 2

• Last lecture of the term.
• Last tutorial today.
• Final examination scheduled for Friday, August 6 from 9:00 to 12:00 Noon.

• Open book exam: text, lecture notes, notes from the ECE 309 Web site, and calculator. Five questions of equal value. There will be at least one heat transfer question.

• SLOT/CV

$$\dot{\mathcal{P}}_S = \frac{\partial}{\partial t} S_{CV} - \dot{S}_{in} + \dot{S}_{out} + \sum_{i=1} \left(\frac{\dot{Q}_i}{T_i} \right)_{CV}$$

where

$$\frac{\partial}{\partial t} S_{CV} = \frac{\partial}{\partial t} \int_{CV} s \rho d\mathcal{V}$$

and

$$\dot{S}_{in} = \int_{A_{in}} s \rho \vec{V} \cdot \vec{n} \, dA \quad \text{and} \quad \dot{S}_{out} = \int_{A_{out}} s \rho \vec{V} \cdot \vec{n} \, dA$$

• Alternative Form of SLOT/CV

$$\left(\frac{dS}{dt} \right)_{CV} = \left(s\dot{M} + \frac{\dot{Q}}{T} \right)_{in} - \left(s\dot{M} + \frac{\dot{Q}}{T} \right)_{out} + \dot{\mathcal{P}}_S$$

• Example of a SSSF System
• Boiler and Turbine. Find $\dot{W}_{max}$
• Continuity Equation (Conservation of Mass)

$$\dot{M}_{in} = \dot{M}_{out} = \dot{M}$$

• Define Control Volume with inflows and outflows.
• FLOT/CV

$$\left(\frac{dE}{dt} \right)_{CV} = \dot{Q} - \dot{W} + \left(\dot{M} h \right)_{in} - \left(\dot{M} h \right)_{out}$$

and

$$\left(\frac{dE}{dt}\right)_{CV} = 0$$

• SLOT/CV

$$\left(\frac{dS}{dt}\right)_{CV} = \left(\dot{M}s + \frac{\dot{Q}}{T_R}\right)_{in} - \left(\dot{M}s\right)_{out} + \dot{\mathcal{P}}_S$$

and

$$\left(\frac{dS}{dt}\right)_{CV} = 0$$

Solving for $\dot{Q}$ we find

$$\dot{Q} = \left[\left(\dot{M}s\right)_{out} - \left(\dot{M}s\right)_{in} - \dot{\mathcal{P}}_S\right] T_R$$

Substitute into FLOT equation and solve for $\dot{W}$

$$\dot{W} = \left\{\dot{M}\left(h - T_R s\right)_{in} - \dot{M}\left(h - T_R s\right)_{out}\right\} - T_R \dot{\mathcal{P}}_S$$

• Definition of Specific Availability b

$$b = h - T_R s$$

Now

$$\dot{W} = \dot{M}(b_{in} - b_{out}) - T_R \dot{\mathcal{P}}_S \quad \text{and} \quad \dot{\mathcal{P}}_S \geq 0$$

Therefore

$$\dot{W} \leq \dot{M}(b_{in} - b_{out})$$

• Maximum Power $\dot{W}_{max}$ occurs when $\dot{\mathcal{P}}_S = 0$

$$\dot{W}_{max} = \dot{M}(b_{in} - b_{out})$$

• $b = h - T_R s$ is also called the Exergy.

• A 2T Heat Engine

– undergoes a cycle

– receives Q_A from heat source at temperature T_A

– rejects heat Q_B to a heat sink at temperature T_B

– output is work W

• Energy Conversion Efficiency η

$$\eta = \frac{W(\text{work out})}{Q_A(\text{input})} < 1$$

or

$$\eta = \frac{\dot{W}(\text{power out})}{\dot{Q}_A(\text{input rate})} < 1$$

- Carnot Efficiency

– Steady-state

– Zero inflow and outflow: $M_{in} = M_{out} = 0$

– T_A and T_B are constant

- Apply FLOT/CV and SLOT/CV
- FLOT/CV

$$\left(\frac{dE}{dt}\right)_{CV} = \dot{Q}_A - \dot{Q}_B - \dot{W} + \left(\dot{M}h\right)_{in} - \left(\dot{M}h\right)_{out}$$

However

$$\left(\frac{dE}{dt}\right)_{CV} = 0, \quad \left(\dot{M}h\right)_{in} = 0, \quad \left(\dot{M}h\right)_{out} = 0$$

and

$$\dot{W} = \dot{Q}_A - \dot{Q}_B$$

- SLOT/CV

$$\left(\frac{dS}{dt}\right)_{CV} = \left(\dot{M}s + \frac{\dot{Q}_A}{T_A}\right)_{in} - \left(\dot{M}s + \frac{\dot{Q}_B}{T_B}\right)_{out} + \dot{\mathcal{P}}_S$$

However

$$\left(\frac{dS}{dt}\right)_{CV} = 0, \quad \left(\dot{M}s\right)_{in} = 0, \quad \left(\dot{M}s\right)_{out} = 0$$

Therefore

$$\frac{\dot{Q}_A}{T_A} - \frac{\dot{Q}_B}{T_B} + \dot{\mathcal{P}}_S = 0$$

and $\dot{\mathcal{P}}_S \geq 0$.

- Assume Reversible Heat Engine: $\dot{\mathcal{P}}_S = 0$
- Heat Rejection to Heat Sink

$$\dot{Q}_B = \dot{Q}_A \frac{T_B}{T_A}$$

- Carnot Efficiency

$$\eta = \frac{\dot{W}}{\dot{Q}_A} = \frac{\dot{Q}_A - \dot{Q}_B}{\dot{Q}_A} = 1 - \frac{\dot{Q}_B}{\dot{Q}_A} = 1 - \frac{T_B}{T_A} < 1$$

• Carnot efficiency is upper limit on efficiency of real heat engines.

• Example: If heat is rejected to environment at $T_b = 285\,K$, and if the heat source is combustion gases at $T_A = 2000\,K$, calculate the Carnot efficiency.

$$\eta = 1 - \frac{285}{2000} = 0.858$$

• Metalurical limits the source temperature to $T_A = 1000\,K$, now calculate the Carnot efficiency.

$$\eta = 1 - \frac{285}{1000} = 0.715$$

Most modern power plants operate at efficiencies between $40 - 50\%$.
