ECE 309 M.M. Yovanovich

Week 3

Lecture 1

• General solution of one-dimensional Poisson equation for plane wall, long circular cylinder, and solid sphere.

• Results are presented for:
(i) Temperature distribution
(ii) Wall or Surface Temperature Drop: $T_s - T_f$
(iii) Centerline or Axis Temperature Drop: $T_{\max} - T_f$
(iv) Solid Temperature Drop: $T_{\max} - T_s$
(v) Ratio of Solid to Film Temperature Drops:

$$\frac{\Delta T_{\text{solid}}}{\Delta T_{\text{film}}} = \frac{T_{\max} - T_s}{T_s - T_f} = \frac{1}{2}\frac{hb}{k} = \frac{1}{2}Bi$$

where $Bi = hb/k$ is the Biot number, a dimensionless group.

• See ECE 309 Web site for details. The lecture notes are also available in DC Library.

Lecture 2

Makeup Lecture 1. 10:30-12:00 noon.

• Examples of application of general solution for Poisson equation.

• One-dimensional Poisson's equations and their solutions for plane wall, long solid cylinder, and solid sphere.

• Ohmic heating in a long circular wire of length L, cross-section A_c, electrical resistivity ρ_e, current flow i, current density $J = i/A_c$. Derivation of relation for volumetric heat source strength:

$$\mathcal{P} = J^2 \rho_e$$

• Poisson's Equation with Ohmic Heating

$$\nabla^2 T = -\frac{\mathcal{P}}{k} = -\frac{J^2 \rho_e}{k}$$

• Consult the ECE 309 Web site for details.

• Example of Application of Poisson's Equation.

• Long hollow copper cykinder of inner and outer diamters: $D_i = 13\,mm, D_o = 50\,mm$, and thermal conductivity $k = 381\,W/m \cdot K$. The electrical resistivity of the copper is $2 \times 10^{-8}\,\Omega - m$ and the current density is $J = 5000\,\mathrm{amperes}/cm^2$. The inner surface at $r = a = D_i/2$ is at temperature $T_1 = 26\,°C$ and the outer surface surface at $r = b = D_o/2$ is at temperature $T_2 = 40\,°C$. The temperature distribution is steady-state, i.e. $T(r)$. The governing ODE is

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = -\frac{\mathcal{P}}{k}, \qquad a < r < b$$

• (a) Obtain the temperature distibution $T(r)$.

• (b) Obtain relation of location $r_{\max}$ where the maximum temperature $T_{\max}$ occurs.

• (c) What is the maximum temperature?

• (d) Determine the heat flow rates out of the cylindrical wall through the inner and outer surfaces: $\dot{Q}_{r=a}$ and $\dot{Q}_{r=b}$.

• (a) Temperature distribution is

$$T(r) = -\frac{Pr^2}{4k} + C_1 \ln r + C_2, \qquad a < r < b$$

Apply the boundary conditions to get two relations for C_1 and C_2

$$(1) \qquad T_1 = -\frac{Pa^2}{4k} + C_1 \ln a + C_2$$

and

$$(2) \qquad T_2 = -\frac{Pb^2}{4k} + C_1 \ln b + C_2$$

Solve for C_1 and C_2.

• (b) The location $r_{\max}$ of $T_{\max}$ occurs where

$$\frac{dT}{dr} = 0 \qquad \text{or} \qquad -\frac{Pr}{2k} + \frac{C_1}{r} = 0$$

Solve to get

$$r_{\max} = \sqrt{\frac{2kC_1}{P}}$$

• (c) For given system parameter values

$$r_{\max} = 19.4\, mm \qquad \text{and} \qquad T_{\max} = 41.9\,^{\circ}C$$

• (d) Heat flow rate through inner and outer surfaces per unit length of cylinder.

$$Q_{r=a} = +2\pi a(1)\left(\frac{dT}{dr}\right)_{r=a} = 52,220\, W$$

and
• Heat flow rate through inner and outer surfaces per unit length of cylinder.

$$Q_{r=b} = -2\pi b(1)\left(\frac{dT}{dr}\right)_{r=b} = 39,318\, W$$

• Volumetric heat source strength due to Ohmic heating

$$\mathcal{P} = J^2 \rho_e = (5000 \times 100^2)^2 \times (2 \times 10^{-8}) = 5 \times 10^7\, \frac{W}{m^3}$$

Lecture 3

• Hand out Project 1. Due date is Friday, May 28.

• Single tutorial on June 9 will be held in room CPH 3388. Tutor will be Edward Chan.

• Extended surfaces (fins. See ECE 309 Web site for details.

• Discussed the derivation of ODE for constant cross-section fins of circular or rectangular shape. The geometric parameters are i) conduction area A ii) perimeter P and iii) fin length L. The fin end at the base $x = 0$ is in mechanical contact and the other end $x = L$ is convectively cooled. The lateral boundaries are convectively cooled. The three coefficients are: h_c, the contact conductance,

h the heat transfer coefficient along the sides, and h_e the heat transfer coefficient h_e at the fin end. The base temperature is T_f and the fluid temperature is T_f.

- Effective fin thickness is defined as $t_e = A/P$. If $Bi = ht_e/k < 0.2$, assume the temperature distribution along the fin is one-dimensional, i.e. $T(x)$.

- Introduce the temperature excess: $\theta(x) = T(x) - T_f$. Note that

$$\frac{d\theta}{dx} = \frac{d(T(x) - T_f)}{dx} = \frac{dT}{dx} - \frac{dT_f}{dx} = \frac{dT}{dx} \qquad \text{for} \qquad T_f = \text{constant}$$

- Apply conservation of energy principle to differential control volume.

- Ordinary differential equation for fin is

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0, \qquad 0 < x < L$$

with fin parameter

$$m^2 = \frac{hP}{kA} \qquad \text{therefore} \qquad m = \sqrt{\frac{hP}{kA}}$$

- Solution of ODE is

$$\theta(x) = C_1 \cosh mx + C_2 \sinh mx$$

Lecture 4

- Continuation of fin analysis. Refer to previous lecture material.

- Boundary conditions.

$$x = 0, \qquad \frac{d\theta}{dx} = -\frac{h_c}{k}\left[\theta_b - \theta(0)\right]$$

and

$$x = L, \qquad \frac{d\theta}{dx} = -\frac{h_e}{k}\theta(L)$$

where $\theta_b = T_b - T_f$. See the ECE 309 Web site for details of the solution.

- Fin Heat Transfer Rate

$$\dot{Q}_{\text{fin}} = -kA\left(\frac{d\theta}{dx}\right)_{x=0} = -kA\,m\,C_2$$

• Fin Resistance

$$R_{\text{fin}} = \frac{\theta_b}{\dot{Q}_{\text{fin}}}$$

Consult the material on the ECE 309 Web site for details.

• Fin resistances for several special cases.
(i) Perfect contact at fin base and end cooling.
(ii) Perfect contact at fin base and adiabatic fin tip.
(iii) Perfect contact at fin base and *infinitely long* fin.

• Criterion for infinitely long fin.

$$\text{If} \quad L \geq L_c = 2.65\sqrt{\frac{kA}{hP}}$$

fin is modelled an infinitely long. Can assume that fin tip is adiabatic.

• Fin resistance for perfect contact at base and adiabatic end.

$$R_{\text{fin}} = \frac{1}{\sqrt{(hPkA)} \tanh mL}$$

• Short Fin Effective Length. If $L < L_c$, end cooling is important, then use effective fin length defined as

$$L_{\text{eff}} = L + \frac{A}{P}$$

in the fin resistance relation. For a circular fin $A/P = D/4$.

• Fin Efficiency

$$\eta = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{ideal}}} < 1$$

where $\dot{Q}_{\text{ideal}} = hPL\theta_b$. The entire fin from base to tip is isothermal at the base temperature, i.e $\theta(x) = \theta_b$.

• Demonstrate how the results for a single fin can be used to model a heat sink.

• A base plate has dimensions: $H = 30\,mm, W = 30\,mm, t = 5\,mm$. One face is in contact with thermal sources through a contact conductance of $h_c = 10,000\,W/(m^2 \cdot K)$. The opposite face and the four edges are convectively cooled by air at temperature $25\,°C$ through a uniform heat transfer coefficient $h =$

$15\,W/(m^2 \cdot K)$. The baseplate is an aluminum alloy whose thermal conductivity is $k = 180\,W/(\cdot K)$.

• Compute the total thermal resistance of the base plate including the thermal contact resistance.

• The heat sink consists of the base plate with 100 aluminum alloy pin fins attached to the non-contact face. The pin fins are identical having diameter $D = 1.5\,mm$ and length $L = 20\,mm$. The pin fins are in perfect contact with the base plate, and the lateral surface and the fin tip are convectively cooled through a heat transfer coefficient $h = h_e = 15\,W/(m^2 \cdot K)$. The base plate surface between the pin fins is also convectively cooled with the same heat transfer coefficient $h = 15\,W/(m^2 \cdot K)$.

• Calculate the thermal resistance of the heat sink.

• What is the heat transfer rate for (i) the base plate and (ii) the heat sink if the thermal source temperature is $T_{\text{source}} = 75\,°C$.
