Week 5

Lecture 1

• Convection Heat Transfer. See Website for material on convective heat transfer.

• Overview of Convective Heat Transfer

Newton's Law of Cooling: $\boxed{\dot{Q}_{conv} = h\, A\, (T_w - T_f)}$

A is total convective area, T_w is surface temperature, T_f or T_∞ is fluid temperature, h is heat transfer coefficient whose units are $[W/m^2 \cdot K]$; h is complex parameter which depends on geometry, thermal and fluid properties, fluid flow and boundary conditions.

• Geometry: flat plate, circular cylinder, sphere, spheroids, other shapes; size, aspect ratio (thin or thick), orientation (vertical or horizontal).

• Type of flow: forced, natural of mixed (combination of forced and natural); laminar, turbulent (transitional); developing, fully-developed; steady or transient; internal or external; enclosure.

• Boundary condition: (i) isothermal wall (T_w = constant); or (ii) isoflux wall (q_w = constant).

• Type of fluid: viscous oil; water; gases (air); liquid metals.

• Fluid properties: symbols and units:
mass density: ρ $[kg/m^3]$
specific heat capacity: c_P $[J/kg \cdot K]$
dynamic viscosity: μ $[N \cdot s/m^2]$
kinematic viscosity: $\nu = \mu/\rho$ $[m^2/s]$
thermal conductivity: $k_f\, [W/m \cdot K]$
thermal diffusivity: $\alpha = k_f/(\rho\, c_p)$ $[m^2/s]$
isobaric compressibility: β $[1/K]$.
All properties are temperature dependent. They are usually determined at the film temperature defined as the average of the wall temperature and the fluid

free stream temperature: $T_{film} = (T_w + T_\infty)/2$. Properties of many fluids: (gases, liquids and liquid metals) are presented in Handbooks and in Appendices in Text.

• Dimensionless Groups in Correlation Equations:

• Prandtl number:$Pr = \nu/\alpha$ where $0 < Pr < \infty$;

• Forced flow: Reynolds number: $Re = \rho U \mathcal{L}/\mu = U\mathcal{L}/\nu$

• Peclet number: $Pe = U\mathcal{L}/\alpha = Re\, Pr$; U is a velocity scale, $\mathcal{L}$ is a length scale.

• Natural convection: Grashof number: $Gr = g\beta(T_w - T_f)\mathcal{L}^3\rho^2/\mu^2 = g\beta(T_w - T_f)\mathcal{L}^3/\nu^2$

• Rayleigh number: $Ra = Gr\, Pr = g\beta(T_w - T_f)\mathcal{L}^3/(\alpha \cdot \nu)$

• Nusselt number:$Nu = h\mathcal{L}/k_f$

• Stanton number: $St = h/(U\,\rho\, c_p) = Nu/(Re\, Pr)$.

• Correlation equations for boundary layer flow:

• Forced Convection:
$Nu = f(Re, Pr)$ or $Nu = C_w\, Re^m\, Pr^n$, C_w depends on geometry, type of flow, boundary condition and choice of $\mathcal{L}$, index m depends on type of flow (laminar or turbulent), index n depends on type of fluid and type of flow

• Natural convection:
$Nu = f(Gr, Pr)$ or $Nu = C_w\, Ra^m\, Pr^n$; C_w depends on geometry, type of flow, boundary condition and choice of $\mathcal{L}$, index m depends on type of flow (laminar or turbulent), index n depends on type of fluid and type of flow.

• Diffusive Limit:
For small Reynolds and Grashof numbers, use the relation:
$Nu = (Nu^p_{\text{diffusion}} + Nu^p_{\text{BL}})^{1/p}$
p is empirical parameter to give good agreement between correlation and data.

• Typical ranges of convective heat transfer coefficients. See material on Web site.

Lecture 2

Some Correlation Equations.

• Local $h(x)$ and average h heat transfer coefficients for forced, laminar flow over a plate.

• Average Heat Transfer Coefficient and Nusselt Number

$$h = \frac{1}{L}\int_0^L h(x)dx \qquad \text{and} \qquad Nu_L = \frac{hL}{k_f}$$

• Isothermal (UWT) Plate. $T_w = \text{const.}$ and $q_w = f(x)$.

$$\frac{h(x)x}{k_f} = c_w \left(\frac{Ux}{\nu}\right)^{1/2} \left(\frac{\nu}{\alpha}\right)^{1/3}$$

or as

$$Nu_x = \frac{0.3387\, Re_x^{1/2} Pr^{1/3}}{[1 + (0.0468/Pr)^{2/3}]^{1/4}}$$

with restrictions: $100 < Re_x < 500,000$ for laminar flow, and all fluids: $0 < Pr < \infty$.

• For air ($Pr = 0.71$)

$$Nu_x = 0.291\, Re_x^{1/2} \qquad \text{and} \qquad Nu_L = 0.582\, Re_x^{1/2}$$

• Isoflux (UWF) Plate. $q_w = \text{const.}$ and $T_w = f(x)$.

$$\frac{h(x)x}{k_f} = c_w \left(\frac{Ux}{\nu}\right)^{1/2} \left(\frac{\nu}{\alpha}\right)^{1/3}$$

or as

$$Nu_x = \frac{0.4637\, Re_x^{1/2} Pr^{1/3}}{[1 + (0.0205/Pr)^{2/3}]^{1/4}}$$

with restrictions: $100 < Re_x < 500,000$ for laminar flow, and all fluids: $0 < Pr < \infty$.

• For air ($Pr = 0.71$)

$$Nu_x = 0.404\, Re_x^{1/2} \qquad \text{and} \qquad Nu_L = 0.808\, Re_x^{1/2}$$

Lecture 3

• Simple, useful relations for air cooling.

• For UWT, laminar flow

$$h = 3.78\sqrt{\frac{U}{L}} \quad \text{at} \quad T_{film} = 300\,K$$

and

$$h = 3.88\sqrt{\frac{U}{L}} \quad \text{at} \quad T_{film} = 350\,K$$

Observe the relatively small difference in the coefficients 3.78 and 3.88 at these two film temperatures.

• Newton's Law of Cooling for UWT Plate of Area $A = LW$ at $T_{film} = 350\,K$.

$$\dot{Q}_{conv} = hA(T_w - T_\infty) = 3.888\sqrt{\frac{U}{L}}LW(T_w - T_\infty)$$

This is a convenient relation for many thermal analyses.

$$\dot{Q}_{conv} = f \times (\text{Power Dissipation}) \quad \text{where} \quad 0 < f < 1$$

• For UWF, laminar flow

$$h = 5.32\sqrt{\frac{U}{L}} \quad \text{at} \quad T_{film} = 300\,K$$

and

$$h = 5.30\sqrt{\frac{U}{L}} \quad \text{at} \quad T_{film} = 350\,K$$

Observe the very small difference in the coefficients 5.32 and 5.30 at these two film temperatures.

• Newton's Law of Cooling for UWT Plate of Area $A = LW$ at $T_{film} = 350\,K$.

$$\dot{Q}_{conv} = hA(T_w - T_\infty) = 5.30\sqrt{\frac{U}{L}}LW(\bar{T}_w - T_\infty)$$

where the average wall temperature is used:

$$\bar{T}_w = \frac{1}{L}\int_0^L T_w(x)dx$$

This is a convenient relation for many thermal analyses.

$$\dot{Q}_{conv} = f \times (\text{Power Dissipation}) \quad \text{where} \quad 0 < f < 1$$

• Reynolds Number at $300\,K$ and $350\,K$

$$Re_L = \frac{UL}{\nu} = 62,850\,UL \quad \text{at} \quad 300\,K$$

and

$$Re_L = \frac{UL}{\nu} = 47,850\,UL \quad \text{at} \quad 350\,K$$

• Natural Convection from Vertical Isothermal (UWT) Plate. Laminar Flow.

$$\frac{hL}{k_f} = c_w \left(\frac{g\beta(T_w - T_\infty)L^3}{\nu^2} \right)^{1/4} \left(\frac{\nu}{\alpha} \right)^{1/4}$$

or in the compact form:

$$Nu_L = c_w Gr_L^{1/4} Pr^{1/4}$$

For gases $\beta = 1/T_\infty$. Alternative form of correlation equation is

$$Nu_L = c_w Ra_L^{1/4} \quad \text{where} \quad Ra_L = Gr_L Pr = \frac{g\beta(T_w - T_\infty)L^3}{\alpha\nu}$$

• Correlation Equation for UWT, Vertical Plate. Laminar Flow.

$$Nu_L = \frac{0.670\, Ra_L^{1/4}}{\left[1 + (0.5/Pr)^{9/16}\right]^{4/9}}$$

with restrictions:

$$10^4 < Gr_L < 10^8 \quad \text{and} \quad 0 < Pr < \infty$$

• Simple, useful relations for UWT, Laminar Flow of Air at $T_{film} = 300\,K$ and $T_{film} = 350\,K$

$$h = 5.48 \left(\frac{T_w - T_\infty}{LT_\infty} \right)^{1/4} \quad \text{at} \quad T_{film} = 300\ K$$

and

$$h = 5.45 \left(\frac{T_w - T_\infty}{LT_\infty} \right)^{1/4} \quad \text{at} \quad T_{film} = 350\ K$$

Note the negligible difference in the coefficients for the two film temperatures.

• Newton's Law of Cooling for Vertical UWT Plate

$$\dot{Q}_{conv} = hA(T_w - T_\infty) = 5.45 \left(\frac{T_w - T_\infty}{LT_\infty} \right)^{1/4} LW\,(T_w - T_\infty)$$

This is a convenient relation for many thermal analyses. Note that:

$$\dot{Q}_{conv} = f \times (\text{Power Dissipation}) \quad \text{where} \quad 0 < f < 1$$

• Types of Forced and Natural Convection Problems which can be solved. Here are a few examples.

Given	Given	Given
T_w	$\dot{Q}_{conv}$	$\dot{Q}_{conv}$
T_∞	T_∞	T_∞
L	L	T_w
W	W	U
U	U	W
Find	Find	Find
$\dot{Q}_{conv}$	T_w	L
