Week 6

Lecture 1

• Natural convection from isothermal (UWT) long horizontal circular cylinder of diameter D.
• General boundary layer correlation equation. See Table C.8 of the text.

$$Nu_D = C\,(Gr_D Pr)^n = C\,(Ra_D)^n$$

where the definitions of the Nusselt (Nu_D), Grashof (Gr_D), and Rayleigh (Ra_D) numbers are:

$$Nu_D = \frac{hD}{k_f}, \qquad Gr_D = \frac{g\beta(T_w - T_\infty)D^3}{\nu^2}, \qquad Ra_D = Gr_D Pr = \frac{g\beta(T_w - T_\infty)D^3}{\alpha\nu}$$

and for ideal gases (approximation for air):

$$\beta = \frac{1}{T_\infty}$$

For air, $Pr = 0.71$.

Table 1: Natural convection from horizontal isothermal circular cylinder

$Ra_D = Gr_D Pr$	C	n	Flow
$10^3 - 10^9$	0.53	1/4	Laminar
$10^9 - 10^{12}$	0.13	1/3	Turbulent

• Natural convection from vertical isothermal plate of height L and width W. Area is $A = LW$. Boundary layer correlation equation is

$$Nu_L = C(Gr_L Pr)^n = C\,Ra_L^n$$

The gravity vector is parallel to the side of length L. The Nusselt (Nu_L), Grashof (Gr_L) and Rayleigh (Ra_L) numbers are defined as

$$Nu_L = \frac{hL}{k_f}, \qquad Gr_L = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2}, \qquad Ra_L = Gr_L Pr = \frac{g\beta(T_w - T_\infty)L^3}{\alpha\nu}$$

and for ideal gases (approximation for air):

$$\beta = \frac{1}{T_\infty}$$

For air, $Pr = 0.71$.

Table 2: Natural convection from vertical isothermal plate

$Ra_L = Gr_L Pr$	C	n	Flow
$10^5 - 10^9$	0.555	0.25	Laminar
$> 10^9$	0.021	0.4	Turbulent

All fluid properties are evaluated at the film temperature: $T_{film} = (T_w + T_\infty)/2$. See Tables C.5a (English Units) and C.5b (SI units) for properties of air.

Lecture 2

- Forced Convection Correlation Equations
- Long, Isothermal (UWT) Circular Cylinder in Cross Flow. Churchill and Bernstein (1977). $L/D > 100$

$$Nu_D = S_D^\star + \frac{0.62\, Re_D^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

This correlation equation is based on the average value of the heat transfer coefficient. It is valid for laminar and turbulent flow.
The diffusive term for $Re_D \to 0$ is obtained from two relations:

$$S_D^\star = \frac{4}{\pi}\left(\frac{1 + 0.869(L/D)^{0.76}}{0.5 + L/D}\right), \qquad 0 \le \frac{L}{D} \le 8$$

and

$$S_D^\star = \frac{4}{\sqrt{\pi}} \frac{1}{\sqrt{(1 + 0.5D/L)}} \frac{1}{\ln(2L/D)}, \qquad \frac{L}{D} > 8$$

The Nusselt (Nu_D) and Reynolds (Re_D) numbers are defined as

$$Nu_D = \frac{hD}{k_f} \qquad \text{and} \qquad Re_D = \frac{UD}{\nu}$$

Restrictions on the correlation equation:

$$Re_D Pr > 0.2, \qquad 0 < Pr < \infty, \qquad 0 < Re_D < 10^7$$

All fluid properties are evaluated at the film temperature: $T_{film} = (T_w + T_\infty)/2$.

• Forced convection from an isothermal (UWT) sphere of diameter D. Whitaker (1972).

$$Nu_D = S_D^\star + \left(0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right) Pr^{0.4}$$

which is valid for laminar and turbulent flow. The diffusive term for $Re_D \to 0$ is $S_D^\star = 2$.

Restrictions on the correlation equation:

$$0.70 < Pr < 380 \qquad \text{and} \qquad 0 < Re_D < 7.6 \times 10^4$$

• General correlation for forced convection from isothermal (UWT) convex geometries developed by Yovanovich (1988). The correlation is based on $\mathcal{L} = \sqrt{A}$ where A is the total heat transfer surface.

$$Nu_{\mathcal{L}} = S_{\mathcal{L}}^\star + \left[0.15 \left(\frac{P}{\mathcal{L}}\right)^{1/2} Re_{\mathcal{L}}^{1/2} + 0.35\, Re_{\mathcal{L}}^{0.566}\right] Pr^{1/3}$$

This correlation is applicable for laminar and turbulent flow. P represents the maximum perimeter of the geometry (it depends on the flow direction). The diffusive term for $Re_D \to 0$ does not depend on the flow direction, and it lies in the range: $3.19 \le S_{\mathcal{L}}^\star \le 4.4$. The lowest value corresponds to a thin circular disk and the largest value corresponds to a long, circular cylinder where $L/D = 10$. For most three-dimensional geometries use the approximation

$$S_{\mathcal{L}}^\star = 3.54$$

Correlation equation was developed for axisymmetric flow over isothermal spheroids such as sphere, oblate (flattened sphere such as the earth) and prolate (such as a football) spheroids. The fluid flows along the minor axis for the oblate, and along the major axis for the prolate.

Restrictions on correlation equation.

$$Pr > 0.71 \qquad \text{and} \qquad 0 < Re_{\mathcal{L}} < 10^5$$

Nortel Networks is testing the limits of application of the correlation equation for cuboids.

• Application to a finite circular cylinder of length L and diameter D whose total heat transfer area is

$$A = \pi DL + 2\,\frac{\pi\, D^2}{4}$$

For axial flow $P = \pi D$, and for cross-flow, $P = 2(D + L)$. This parameter accounts for flow direction.

Lecture 3

• Radiation: Chapter 14, Section 14.8. Also see ECE 309 Web site for additional material.

• Planck's Distribution Law
• Wien's Displacement Law
• Stefan-Boltzmann Law of Radiation: $E_b = \sigma T^4$
• Stefan-Boltzmann Constant: $\sigma = 5.67 \times 10^{-8}\, W/(m^2 \cdot K^4)$
• Real surface radiation: $E = \epsilon \sigma T^4$ with $0 < \epsilon < 1$
• Absorptivity α, Reflectivity ρ, Transmissivity τ
• For a layer of some substance: $\alpha + \rho + \tau = 1$
• For liquids and solids: $\tau = 0$, and $\alpha + \rho = 1$
• For light gases such as hydrogen, oxygen, nitrogen and mixtures (dry air), $\tau = 1$
• For complex gases such as water vapor and CO, $\tau \neq 1$
• Specular relections (smooth, clean surfaces such as a mirror) incident and relected angles are equal: $\theta_i = \theta_r$
• Diffuse reflections (rough, dirty, oxidized surfaces), reflected radiation goes in all directions (no preferential direction)
• Kirchhoff's Law: $\epsilon_\lambda = \alpha_\lambda$ at thermal equilibrium
• Gray surface: use ϵ independent of wavelength
• Radiation view factors for two isothermal surfaces A_1 and A_2: $0 \leq F_{12} \leq 1$ and $0 \leq F_{21} \leq 1$. These are dimensionless parameters.
• Reciprocity relation: $A_1 F_{12} = A_2 F_{21}$. See notes for definitions.
