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Flow Work

Consider mass flow into a control volume through an inlet of constant cross-section during the time interval Δt.

Continuity Equation
The mass inflow into the CV during the time interval Δt is

$$\Delta M_{\text{in}} = \rho_{\text{in}}\, A_{\text{in}}\, \bar{V}_{\text{in}}\, \Delta t \qquad [kg]$$

Flow Work
The flow work of the fluid entering the CV is defined as:

$$\Delta W_{\text{in}} = (\text{Force})\ \text{X}\ (\text{Displacement}) = (P_{\text{in}}\, A_{\text{in}}) \times \left(\bar{V}_{\text{in}}\, \Delta t\right)$$

Applying the continuity equation we have

$$\Delta W_{\text{in}} = \frac{P_{\text{in}}\, \Delta M_{\text{in}}}{\rho_{\text{in}}} = P_{\text{in}}\, v_{\text{in}}\, \Delta M_{\text{in}} = (P\, v\, \Delta M)_{\text{in}}$$

where $v = 1/\rho$. Similarly at the outlet we have

$$\Delta W_{\text{out}} = (P\, v\, \Delta M)_{\text{out}}$$

In general we can express the time rate of flow work out of the CV as

$$\dot{W}_{\text{out}} = \iint_{\text{A}} P\, v\, \rho\, \vec{V} \cdot \vec{n}\, dA$$

Net Inflow Rate of Energy and Work Into CV
The energy and work associated with mass flow across control surfaces can be determined from the following general expression:

$$\dot{E}_{\text{CV}} + \dot{W}_{\text{CV}} = -\iint_{\text{CS}} (e + P\, v)\ \rho\, \vec{V} \cdot \vec{n}\, dA$$