PROJ1S99.TEX

UNIVERSITY OF WATERLOO
DEPARTMENT OF MECHANICAL ENGINEERING

E&CE 309 Thermodynamics I

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Project No. 1, Part 1. This part of Project No. 1 must be handed in at the start of the next lecture, Friday, May 28. You may consult the TAs and you may discuss the project with other students, but you must be the sole contributor to the material which you hand in. Part 2 of Project No. 1 will be given to you during the lecture hour on Friday, May 28. You will be given 15 minutes to work on Part 2. It must be handed in at the end of that lecture hour. Part 1 will count 8 points and Part 2 will count 2 point.

A hot fluid is contained in a spherical shell of inner radius r_i and outer radius r_o. The thermal conductivity of the spherical wall is k, which is assumed to be constant. The temperature of the hot fluid is T_{f1} and the heat transfer coefficient at the inner boundary is h_1. The temperature of the fluid at the outer boundary is $T_{f2} < T_{f1}$ and the heat transfer coefficient is h_2. Since there are no distributed sources or sinks within the spherical wall, and the temperature is steady-state, i.e. $T = T(r)$, the governing equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \qquad r_i \le r \le r_0$$

(a) Specify the two boundary conditions.

(b) Obtain the temperature distribution within the spherical wall, and put your result in the form:

$$\frac{T_{f1} - T(r)}{T_{f1} - T_{f2}} = ?$$

Give a physical interpretation of the terms which appear in the numerator and denominator of the right-hand side of the solution.

(c) Obtain the expression for the heat transfer rate through the spherical wall using the Fourier Law of Conduction: $\dot{Q} = -k4\pi r^2 \frac{dT}{dr}$.

(d) Use the definition of the total thermal resistance of the system: $R_{\text{total}} = (T_{f1} - T_{f2}) / \dot{Q}$ to demonstrate that the total resistance consists of the sum of the inner film resistance, the shell resistance and the outer film resistance.

(e) Sketch the thermal *circuit* showing clearly the nodes, the thermal resistors and the throughput. Label the nodes and resistors.