UNIVERSITY OF WATERLOO
DEPARTMENT OF MECHANICAL ENGINEERING
ME 353 HEAT TRANSFER 1

December 9, 1996 **Time: 9-12 A.M.**
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Three-hour Closed Book Final Examination. Two crib sheets (both sides) are permitted. All questions are of equal value. State clearly and justify all assumptions made and label all sketches and thermal circuits clearly. Allocate your time wisely and do not spend disproportionate time on any single problem. Good luck.

PROBLEM 1

[20]

A fireclay brick of dimensions $50\ mm \times 80\ mm \times 180\ mm$ is removed from a kiln at the uniform temperature $T_i = 1500\ K$, and it is cooled in air which is at the constant temperature $T_f = 300\ K$. The heat transfer coefficient is assumed to be uniform over all six faces of the brick and its magnitude is $60\ W/m^2 \cdot K$. The thermophysical properties of the brick are: $k = 1.0\ W/m \cdot K, \rho = 2000\ kg/m^3, c_p = 960\ J/kg \cdot K$. The cooling period is $60\ minutes$.

(a) Can the lumped capacitance method be used to calculate the cooling rate?

(b) Compute the temperature at the center of the brick after $60\ minutes$.

(c) Which dimension of the brick controls the cooling process, and explain why?

The first term of the series solution for the dimensionless temperature excess ϕ within a plane wall is given by

$$\phi = A_1 \exp\left(-\delta_1^2 Fo\right) \cos\left(\delta_1 \zeta\right)$$

where the Fourier coefficient is given by

$$A_1 = \frac{2 \sin\left(\delta_1\right)}{\delta_1 + \sin\left(\delta_1\right) \cos\left(\delta_1\right)}$$

and the first eigenvalue which is the root of the characteristic equation: $\delta_1 \sin\left(\delta_1\right) = Bi \cos\left(\delta_1\right)$ can be computed accurately by means of the following approximation:

$$\delta_1 = \frac{\delta_{1,\infty}}{\left[1 + \left(\delta_{1,\infty}/\sqrt{Bi}\right)^n\right]^{1/n}}$$

and $\delta_{1,\infty} = \pi/2$ corresponding to $Bi \to \infty$. The value of the exponent is $n = 2.139$. The dimensionless parameters are the Fourier number Fo, the dimensionless position within the plane wall $0 \le \zeta = x/L \le 1$, and the Biot number $Bi = hL/k$ where L is the half-thickness of the plane wall.

PROBLEM 2

[20]

A $50 - W$ incandescent bulb is placed in a large room and it is cooled by atmospheric air whose velocity is $0.5\ m/s$. Its surface temperature is measured to be $140\ C$ and the air and surrounding temperatures are $25\ C$. The bulb can be modeled as a sphere of diameter $50 - mm$.
You are given the thermophysical properties of the air at the film temperature:
$k = 0.0261\ W/m \cdot K, \nu = 15.71 \times 10^{-6}\ m^2/s, Pr = 0.71$, and the following correlation equations for pure forced and pure natural convection from isothermal spheres:

Rance and Marshall Correlation

$$Nu_D = 2 + 0.6\ Re_D^{1/2} Pr^{1/3}$$

which is valid for $Pr > 0.7$.

Churchill Correlation

$$Nu_D = 2 + F(Pr) Ra_D^{1/4}$$

where the Prandtl number function is defined as

$$F(Pr) = \frac{0.589}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}}$$

which is valid for $Pr > 0.7$ and $Ra_D \leq 10^{11}$. These correlation equations are reported in the 4th edition of the text of Incropera and DeWitt. Use the air properties given above for *both* correlation equations.

(a) Compute the heat transfer rate for pure forced convection Q_{FC}.

(b) Compute the heat transfer rate for pure natural convection Q_{NC}.

(c) Compute the *maximum* radiation heat transfer rate Q_{RAD} from the surface of the bulb to the surroundings.

(d) Use the above results to estimate the conduction heat transfer rate Q_{COND} from the bulb through its support.

PROBLEM 3

[20]

To determine air velocity fluctuations, it is proposed to measure the electric current required to maintain a platinum wire of 0.6 mm diameter at a constant temperature of $117^0 C$ in a stream of air at $37^0 C$.

The air properties at 350 K are:
$k = 0.0300\ W/m \cdot K, c_p = 1009\ J/kg \cdot K, \mu = 208.2 \times 10^{-7}\ N \cdot s/m^2, Pr = 0.700, \rho = 0.9950\ kg/m^3$.

(a) Assuming Reynolds numbers in the range $40 < Re_D < 4000$, *develop* a relationship between the wire current and the velocity of the air which is in cross flow over the wire.

(b) Calculate the current required when the air velocity is 30 m/s and the electrical resistivity of the platinum wire is $16.67 \times 10^{-6}\ \Omega \cdot cm$.

You may use the empirical correlation due to Hilpert developed for long circular cylinders in cross flow:

$$Nu_D = \frac{\overline{h}D}{k_f} = C Re_D^m \, Pr^{1/3}$$

where the correlation coefficients C and m are given below:

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

PROBLEM 4

[20]

Air at $3 \times 10^{-4}\ kg/s$ and $27^0 C$ enters a rectangular duct that is 1 m long and 4 mm by 16 mm in cross-section. A uniform heat flux of 600 W/m^2 is imposed on the duct surface.

The air properties are: $k = 0.0263\ W/m \cdot K, c_p = 1007\ J/kg \cdot K, \mu = 184.6 \times 10^{-7}\ N \cdot s/m^2, Pr = 0.707$.

(a) Calculate the Reynolds for this system.

(b) Calculate the air bulk temperature at the duct outlet by means of an overall energy balance.

(c) Calculate the duct surface temperature at the duct outlet. Use the information given below for fully-developed laminar flow through an isoflux rectangular duct.

The Nusselt number based on the hydraulic diameter D_h for rectangular ducts depends on the aspect ratio b/a of the duct cross-section. The following values are reported:

$\frac{b}{a}$	Nu_{D_h}
1	3.61
2	4.12
3	4.79
4	5.33
8	6.49
∞	8.23

PROBLEM 5

[20]

Consider two large gray diffuse parallel surfaces which are separated by a small gap. The air in the gap is transparent to radiation heat transfer.

(a) If the surface emissivities are $\epsilon_1 = 0.8$ and $\epsilon_2 = 0.9$ respectively, sketch the equivalent thermal circuit labelling clearly all nodes and resistors, and calculate the radiation resistance of the system.

(b) A thin, high conductivity *shield* whose emissivities are $\epsilon_{s,1} = 0.1$ and $\epsilon_{s,2} = 0.2$ is placed in the gap between the two surfaces defined in part (a). Sketch the equivalent thermal circuit labelling clearly all nodes and resistors, and calculate the radiation resistance of this system.

(c) If $\epsilon_{s,1} = \epsilon_{s,2}$, find the value of the shield emissivity which will *reduce* the net radiation heat transfer of part (a) by a factor of 20.