UNIVERSITY OF WATERLOO
DEPARTMENT OF MECHANICAL ENGINEERING
ME 353 HEAT TRANSFER 1

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Two-hour Closed Book Mid-term Examination. You are allowed one crib sheet (both sides). All questions are of equal value. State clearly all assumptions made and label all sketches and thermal circuits clearly.

PROBLEM 1

[10]

A spherical aluminum shell, with an electrical heater located at its center, is used in tests to determine the thermal conductivity of insulating materials. The inner and outer radii of the shell are $r_1 = 0.15\,m$ and $r_2 = 0.18\,m$, respectively. Testing is done under steady-state conditions with the inner surface maintained at $250\,^\circ C$. The thermal conductivity of the aluminum alloy at temperatures near $250\,^\circ C$ is $220\,W/m \cdot K$.
In a particular test, a spherical shell of insulation is cast on the outer surface of the aluminum shell to a thickness of $0.12\,m$. The system is located in a large room where the air temperature is $20^\circ C$, and the combined convection and radiation heat transfer coefficient at the outer surface of the insulation is estimated to be $h_0 = h_{\text{conv}} + h_{rad} = 30\,W/m^2 \cdot K$. The electrical heater produces $80\,W$ under steady-state conditions. The thermal contact resistance between the aluminum and the insulation can be neglected.

(a) Sketch the system.
(b) Show the equivalent thermal circuit, labeling all temperature nodes and thermal resistances.
(c) Calculate the thermal conductivity of the insulation.
(d) Find the temperatures of the inner and outer surfaces of the insulation.
(e) If the emissivity of the outer surface of the insulation is $\epsilon = 0.87$, find the value of h_{rad}.

PROBLEM 2

[10]

A circular metallic rod of diameter, D, and length, L has constant thermal conductivity, k. One end of the rod at $x = 0$ is isothermal at temperature T_0 and the other end at $x = L$ is adiabatic. A portion of the rod, $0 \leq x \leq fL$ where $0 < f < 1$, has adiabatic sides, while the remaining portion of the rod is exposed to a fluid at constant temperature $T_f < T_0$, and there is convective cooling through a uniform film coefficient, h, from the exposed portion. The Biot number $Bi = hD/2k < 0.2$

(a) Sketch the system showing clearly all geometric and thermophysical parameters.
(b) Show the equivalent thermal circuit with its temperature nodes and thermal resistances.
(c) Obtain a relation for the geometric parameter f when the thermal resistance of the adiabatic portion is equal to the convectively cooled portion. Do not attempt to solve the transcendental relation.
(d) Given the values of the parameters: $D = 6\,mm, L = 100\,mm, k = 25\,W/m \cdot K, h = 65\,W/m^2 \cdot K$, obtain a numerical relation for the parameter f. Do not attempt to find the root of the transcendental equation.

PROBLEM 3

[10]

Carbon steel billets of diameter, $D = 150\,mm$ and length, $2L = 150\,mm$ are heat treated in a gas-fired furnace whose gas temperature is maintained at $1200\,K$, and the total heat transfer coefficient is estimated to be $80\,W/m^2 \cdot K$. The billets enter the furnace at a uniform temperature of $300\,K$. The thermophysical properties of the carbon steel (AISI 1010) at approximately $550\,K$ are: $\rho = 7830\,kg/m^3$, $k = 51.2\,W/m \cdot K$, $c_p = 540\,J/kg \cdot K$.

(a) Calculate the Biot numbers: $Bi_1 = hD/(2k), Bi_2 = hL/k, Bi_3 = h\mathcal{L}/k$, where $\mathcal{L} = V/S$; and V is the volume and S is the total active heat transfer surface area.
(b) Calculate the characteristic time for the system, i.e. t_c.
(c) How long must the billets remain in the furnace for the temperature at the origin to reach $800\,K$?.
(d) After reaching a temperature of $800\,K$, the billet is removed from the furnace and allowed to cool by natural convection and radiation to $320\,K$. If the effective heat transfer coefficient, $h_e = h_{\text{conv}} + h_{\text{rad}}$ is taken to be $25\,W/m^2 \cdot K$ during the cooling period, calculate the time required for the cooling process assuming that the ambient and surroundings temperatures are equal to $300\,K$.