Week 12

Lecture 1

See website for Maple worksheets for air properties correlation equations, and natural convection calculations.
Natural convection across annular space bounded by two isothermal horizontal concentric cylinders and two isothermal concentric spheres.
Horizontal circular cylinders: Eqs. (9.58), (9.59) and (9.60).
Concentric spheres: Eqs. (9.62), (9.63) and (9.61).
See Maple worksheets dealing with these systems.
Simpler form of the correlation equations was presented in the lecture.
Heat transfer rate across the space is obtained from Newton's Law of Cooling:

$$Q_{sys} = hA_i(T_i - T_o)$$

where the inner surface area is $A_i = \pi D_i L_{cylinder}$ for the inner cylinder and $A_i = \pi D_i^2$ for the inner sphere, and for both enclosures:

$$h = \frac{k_f Nu_{D_i}}{D_i}$$

Discussion of plot of Nusselt versus Rayleigh number for a wide range of values of Rayleigh number.
For small values of Rayleigh number there is negligible fluid flow and heat transfer across the space is essentially by conduction. Therefore,

$$Nu_{D_i} = \frac{hD_i}{k_f} = S^\star_{D_i}$$

for pure conduction across the space and the dimensionless shape factor is defined as

$$S^\star_{D_i} = \frac{SD_i}{A_i}$$

The dimensionless shape factors for the two systems are obtained from

$$S^\star_{D_i} = \frac{2}{\ln \dfrac{D_o}{D_i}}, \qquad \text{for concentric cylinders}$$

and

$$S^{\star}_{D_i} = \frac{2}{1 - D_i/D_o}, \qquad \text{for concentric spheres}$$

Note that for both systems, as $D_o/D_i \to 1$, $S^{\star}_{D_i} \to \infty$.

For high values of the Rayleigh number the heat transfer is by laminar fluid flow, therefore,

$$Nu_{D_i} = (Nu_{D_i})_{bl}$$

for laminar boundary layer flow over the inner and outer boundaries. For concentric isothermal cylinders and concentric isothermal spheres use the simpler, alternative forms of the Raithby-Hollands correlation equations which do not require the intermediate steps of the original sets of correlation equation presented in Incropera and DeWitt (see Maple worksheets).
For concentric circular cylinders use

$$(Nu_{D_i})_{bl} = 0.772 \left[\frac{Pr}{0.861 + Pr}\right]^{1/4} \frac{Ra_{D_i}^{1/4}}{\left[1 + (D_i/D_o)^{3/5}\right]^{5/4}}$$

and for concentric spheres use

$$(Nu_{D_i})_{bl} = 0.74 \left[\frac{Pr}{0.861 + Pr}\right]^{1/4} \frac{Ra_{D_i}^{1/4}}{\left[1 + (D_i/D_o)^{7/5}\right]^{5/4}}$$

where the Rayleigh number for both systems is defined as

$$Ra_{D_i} = \frac{g\beta(T_i - T_o)D_i^3}{\alpha\nu}$$

and

$$\beta = \frac{1}{T_f}$$

with $T_f = (T_i + T_o)/2$.

Outline natural convection from horizontal isothermal cube.

- $Q = hA(T_w - T_f)$
- $h = \dfrac{k_f Nu_{\sqrt{A}}}{\sqrt{A}}$
- $Nu_{\sqrt{A}} = S^{\star}_{\sqrt{A}} + F(Pr)\, G_{\sqrt{A}}\, Ra_{\sqrt{A}}^{1/4}$

where $S^\star_{\sqrt{A}} = 3.388$, $F(Pr = 0.71) = 0.513$ and $G_{\sqrt{A}} = 0.985$ for an isothermal cube cooled by air. For this correlation equation the Rayleigh number is defined as

$$Ra_{\sqrt{A}} = \frac{g\beta(T_w - T_\infty)\left(\sqrt{A}\right)^3}{\alpha\nu}$$

with $\beta = 1/T_\infty$.

Example: Heat transfer in space between two isothermal spheres.

- System parameters: $D_i = 75\,mm, D_o = 200\,mm, T_i = 360\,K, T_o = 300\,K$

$$Ra_{D_i} = \frac{(9.81)(1/330)(360-300)(75/1000)^3}{(26.88\times10^{-6})(18.86\times10^{-6})} = 1.484\times10^6$$

$$S^\star_{D_i} = \frac{2}{1-75/200} = 3.2$$

$(Nu_{D_i})_{bl} = 15.95$
Since $(Nu_{D_i})_{bl} > S^\star_{D_i}$, use the larger value to find h. Therefore,

$$h = \frac{(0.02856)(15.95)}{75/1000} = 6.074\,W/m^2\cdot K$$

$$Q = (6.074)(\pi)(75/1000)^2(360-300) = 6.44\,W$$

Lecture 2

Radiation. Some material was covered in the first two weeks of the course.
See website for radiation notes and Maple worksheets on view factors.
Chapters 12 and 13.
Section 12.3:
- Planck's Distribution Law, Eq. (12.26)
- Wien's Displacement Law, Eq. (12.27)
- Stefan-Boltzmann Law of Radiation, Eq. (12.28)
- Actual Radiation from isothermal real surfaces. Gray surfaces: emissivity.
- Absorptivity, reflectivity, transmissivity.
- Specular and diffuse relections.
- Kirchhoff's Law.
- Radiation View Factor. Reciprocity Relation.

Lecture 3

- Radiant exchange between two isothermal black surfaces.

- Radiant exchange between two isothermal gray surfaces.
- Two infinite parallel planes.
- Long concentric gray circular cylinders.
- Concentric gray spheres.
- Effect of radiation shields.
