**ME 353 M.M. Yovanovich**

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## Week 4

### Lecture 1

Hand out Project 1. Due Friday, October 16, 12 Noon.

• Extended surfaces or fins; temperature is one-dimensional $T(x)$ when $Bi = ht_e/k < 0.2$ where the effective fin thickness is defined as $t_e = A/P$; where $P$ is the constant fin perimeter and $A$ is the constant fin conduction area.
See Web Site for derivation of general fin equation and general solution for fins with contact conductance, $h_c$, and end cooling, $h_e$;
solution $\theta(x) = T(x) - T_f$, called the temperature excess, is a function of dimensionless parameters: $Bi_c = h_c L/k$, $Bi_e = h_e L/k$, $mL$ with fin parameter: $m = \sqrt{hP/(kA)}$
$P$ is the constant fin perimeter and $A$ is the constant fin conduction area.
• Fin resistance: $\boxed{R_{\text{fin}} = \theta_b/Q}$; $\theta_b = T_b - T_f$.
• Special cases of the general solution:
(a) perfect contact at fin base: $Bi_c = \infty$ and end cooling: $Bi_e > 0$
(b) perfect contact at fin base: $Bi_c = \infty$ and adiabatic end: $Bi_e = 0$;
(c) perfect contact and *infinitely long* fin.
• Criterion for *infinitely long* fin: $L_{crit} = 2.65/\sqrt{(hP/kA)}$
When $L > L_{crit}$, model fin as *infinitely long*, and when $L < L_{crit}$, model fin as finite length with end cooling.

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### Lecture 2

• Show examples of pin fins, straight fins and circular annular fins from telecommunication and microelectronics industries, and automotive industries.
• Sketch the typical straight fin (straight or pin) of length, $L$, conduction area, $A$, and perimeter, $P$.
• End conditions: (i) contact conductance, $h_c$ at the base, $x = 0$, and (ii) convective cooling at the end, $x = L$.
• When $Bi = ht_e/k < 0.2$, then $T(x, y) \to T(x)$.
• Derivation of ODE:
⋆ heat balance on differential control volume (CV) $dV = Adx$:
⋆ conduction into CV at $x$ is $dQ_x = -kAdT/dx$,

$\star$ conduction rate out of CV at $x+dx$ is $Q_x + (dQ_x/dx)\,dx$,
$\star$ convection loss is $dQ_{\text{conv}} = hPdx(T(x) - T_f)$;
$\star$ no sources, steady-state;
$\star$ derive governing second-order ordinary differential equation.
see section 3.6.2 for derivation of ODE and boundary conditions at $x = 0$ (perfect contact) and $x = L$ (convection cooling).

- See Table 3.4 for summary of solutions.
- See Table 3.5 for summary of fin efficiencies: $\eta_f$ for various fin types.

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**Lecture 3**

Review of material on Web site for derivation of general fin equation valid for variable conduction area, $A(x)$, and variable perimeter, $P(x)$; introduce temperature excess: $\theta(x) = T(x) - T_f$; note that $d\theta/dx = dT/dx$ because $T_f$ is constant; consider special case: $A$ and $P$ are constants;
fin equation becomes $\boxed{d^2\theta/dx^2 - m^2\theta = 0}$ in $\boxed{0 < x < L}$ with fin parameter: $\boxed{m = \sqrt{hP/kA}}$ units of $m$ are $1/m$;
at the base, $x = 0$, there is contact conductance, $h_c$, and at the fin end, $x = L$, there is convective cooling, $h_e$;
boundary conditions of the third kind (Robin) are applied at the fin base and fin end:
$\boxed{d\theta(0)/dx = -(h_c/k)\,[\theta_b - \theta(0)]}$ and $\boxed{d\theta(L)/dx = -(h_e/k)\theta(L)}$; $\theta_b = T_b - T_f$;
solution is $\boxed{\theta = C_1 \cosh mx + C_2 \sinh mx}$;
introduce dimensionless fin parameters: $\boxed{Bi_c = h_c L/k}$, $\boxed{Bi_e = h_e L/k}$,
$\boxed{mL = \sqrt{(hP/kA)}\,L}$
solve for the constants of integration which are:

$$C_1 = \frac{\theta_b}{1 + \dfrac{mL\,\phi}{Bi_c}} \quad \text{and} \quad C_2 = -\frac{\theta_b\,\phi}{1 + \dfrac{mL\,\phi}{Bi_c}}$$

where

$$\phi = \frac{mL \tanh mL + Bi_e}{mL + Bi_e \tanh mL}$$

obtain fin heat flow rate: $\boxed{Q_{\text{fin}} = -kA_b d\theta(0)/dx}$; $A_b$ is the base conduction area.
fin resistance: $\boxed{R_{\text{fin}} = \theta_b/Q_{\text{fin}}}$; see Web Site for development of solution and use the Javascript Calculator;
perfect contact at base and end cooling:

$R_{fin} = 1/\sqrt{hPkA}\,\tanh mL$
perfect contact at fin base and *infinitely long* fin:
$R_{fin} = 1/\sqrt{hPkA}$
fin efficiency: $\boxed{\eta_f = Q_{\text{fin}}/Q_{\text{ideal}}}$;
$Q_{\text{ideal}}$ corresponds to an ideal fin whose thermal conductivity is infinitely large
$Q_{\text{ideal}} = \int_0^L hP\theta_b\,dx + h_e A\theta_b = (hPL + h_e A)\theta_b$
special cases of general solution with perfect contact at fin base:
$h_c = \infty$ or $Bi_c = \infty$;
three options at the fin end:
(i) end cooling $h_e > 0$ or $Bi_e > 0$;
(ii) adiabatic end: $h_e = 0$ or $Bi_e = 0$;
(iii) *infinitely long* fin, i.e. $L > L_{crit} = 2.65/\sqrt{hP/kA}$;
see Web Site for several special cases.
Longitudinal fins; pin fins; circular annular fins; analytical solutions for several types of fins; see Table 3.5 for efficiencies of common fin shapes.
Applications of fin solutions:

• example 1 is a circular rod of length $2L$ which connects two walls at temperatures $T_1$ and $T_2$ which are greater than air temperature $T_f$; there is convective cooling from the sides of the rod into the air. Assume perfect contact at the interfaces between the rod and the two walls.
special cases:
(i) when $T_1 = T_2$, the plane of symmetry (adiabatic plane) occurs at mid-point
(ii) when $T_1 > T_2$, the plane of symmetry moves to the right of the mid-point
(iii) when $T_1 < T_2$, the plane of symmetry moves to the left of the mid-point

• example 2 is a system which consists of two finite length fins $L_1, L_2$ with adiabatic ends connected to a rod of length $L_3$ with adiabatic lateral boundaries. Check Biot numbers: is $Bi_1 = h_1 t_e/k < 0.2$ and is $Bi_2 = h_2 t_e/k < 0.2$? Yes.
• system heat transfer rate: $Q_{sys} = (T_{f1} - T_{f2})/R_{sys}$
• system resistance: $R_{sys} = R_{fin1} + R_{rod} + R_{fin2}$
• component resistances:
$R_{fin1} = 1/\sqrt{h_1 PkA}\,\tanh(m_1 L_1)$ and $m_1 = \sqrt{h_1 P/(kA)}$
$R_{fin2} = 1/\sqrt{h_2 PkA}\,\tanh(m_2 L_2)$ and $m_2 = \sqrt{h_2 P/(kA)}$
$R_{rod} = L_3/(kA)$

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