Week 5

Lecture 1
Monday, October 12 lecture cancelled for Thanksgiving Day.

Lecture 2
Resistance of truncated cone of length L, and radii a, b where $b > a$ with restriction: $(b-a)/L << 1$. Thermal conductivity depends on temperature such that $k(T) = k_0(1 + \beta T)$
Lateral boundaries are adiabatic and the ends are isothermal at temperatures: T_1 at $x = 0$ and $T_2 < T_1$ at $x = L$. Assume that $T = T(x)$.
Fourier's Law of Conduction at arbitrary plane x is

$$Q = -k_0(1 + \beta T) A(x) \frac{dT}{dx}$$

Separate variables: T, x and integrate to get:

$$\int_0^L \frac{Q}{A(x)} dx = \int_{T_1}^{T_2} -k_0(1 + \beta\, T)\, dT$$

RHS gives:

$$k_0 \left[1 + \beta \frac{(T_1 + T_2)}{2} \right] (T_1 - T_2) = k_m (T_1 - T_2)$$

where k_m is the value of the thermal conductivity at the arithmetic mean temperature $(T_1 + T_2)/2$. To integrate LHS we require the local conduction area:

$$A(x) = \pi r^2 = \pi \left[a + (b-a) \frac{x}{L} \right]^2$$

Substitute into LHS and integrate. The substitution $u = a + (b-a)x/L$ can be used to simplify the integration. Finally, we find LHS is

$$\int_0^L \frac{Q}{A(x)} dx = \frac{QL}{\pi ab}$$

Interpretation of denominator: πab which is equal to $\sqrt{\pi a^2 \pi b^2}$ which can be expressed as the geometric mean conduction area: $\sqrt{A_i A_o}$ where A_i and A_o are the conduction areas at inlet and outlet of truncated cone.
Resistance is

$$R = \frac{(T_1 - T_2)}{Q} = \frac{L}{k_m \sqrt{A_i A_o}}$$

Discuss Project 1: Problem 3.131 of text. Recommend analysis based on a fin and associated gap. Circular annular fin: see section 3.6.4 of text for equation, solution, heat flow rate, fin efficiency and fin resistance; application to heat sink. See Table 3.5 for expressions for efficiency of several straight fins and pin fins having different cross-sections. See Examples 3.9 and 3.10. See Web Site for Maple solutions for Examples 3.9 and 3.10.

Lecture 3
Project 1 is due Friday, October 16 at noon to Rebecca.

Bounds on resistance of a composite wall. Upper bound is based on parallel adiabats and lower bound is based on parallel isotherms. Actual resistance lies between the upper and lower bound values:

$$R_{LB} \leq R_{\text{actual}} \leq R_{UB}$$

Actual resistance can be estimated by some average value of the upper and lower bounds such as

$$\begin{aligned} R_{\text{approx}} &= \frac{1}{2}(R_{LB} + R_{UB}) && \text{Arithmetic Mean} \\ R_{\text{approx}} &= \sqrt{R_{LB} R_{UB}} && \text{Geometric Mean} \\ \frac{1}{R_{\text{approx}}} &= \frac{1}{2}\left[\frac{1}{R_{LB}} + \frac{1}{R_{UB}}\right] && \text{Harmonic Mean} \end{aligned}$$

When the bounds have relatively close values, the three approximations give close values.

Example: When $R_{LB} = 1$ and $R_{UB} = 2$, then $R_{AM} = 1.5$, $R_{GM} = \sqrt{2} = 1.414$, $R_{HM} = 4/3 = 1.333$. Observe that the R_{GM} value is close to the geometric mean of the R_{AM} and R_{HM} values.
The Geometric Mean value is therefore recommended.

Chapter 4:
- Steady-state, source free, two- and three-dimensional conduction problems.

• Solution methods: (A) analytical; (B) numerical; (C) analog; (D) approximate methods.
analytical: (i) separation of variables; (ii) conformal mapping method; (iii) transform methods such as Fourier sine or cosine, and other transform methods; limited by the geometry and the boundary conditions.
numerical: (i) finite difference method; (ii) finite volume method, (iii) finite element method; (iv) boundary integral equations method; (v) surface element method; (vi) other numerical methods based on some analytical method;
analog: electrical; mass transfer; hydrodynamic;
approximate: upper bound (parallel adiabats) and lower bound (parallel isotherms) on system resistance; flux plotting (based on sketching isotherms and adiabats); approximate methods are cost effective.

• Shape factor S $[m]$. Definitions: $Q = kS(T_1 - T_2) = (T_1 - T_2)/R$
• Relationship between S and R is $S = 1/(kR)$ or $R = 1/(kS)$.
• See Table 4.1 for a short list of some useful shape factors.
• See Maple worksheets on shape factors on ME 353 Website.

Lecture 4
Makeup lecture 2.
Discussion on shape factors given in Table 4.1.
• Sphere buried in semiinfinite region
• Long circular cylinder buried in semiinfinite region
• Long circular cylinder in a square cylinder.

• How to find approximate expression for square cylinder inside a square cylinder using the exact solution for the circle inside a circle; two rule method: (a) set the inner areas equal; (b) set the volumes equal; discuss other rules that may be used to find the outer radius; it was suggested that (i) the outer areas can be set equal; (ii) the minimum spacing can be set equal; two approximations based on analog methods were presented: $\boxed{S/L = 2\pi/(0.93\ln(\beta/\alpha) - 0.0502)}$ for $\beta/\alpha > 1.4$ and $\boxed{S/L = 2\pi/(0.785\ln(\beta/\alpha))}$ for $\beta/\alpha < 1.4$.
