ME 353 M.M. Yovanovich

Week 9

Lecture 1

Makeup lecture 3.
Week 8 lecture summary and several pages of summary of convective heat transfer correlations are available in Engineering Photocopy Center. Pick up and bring to future lectures.

See Appendices for special functions. Table B.2 on page 857 for $erf(w)$ and $0 \leq w \leq 3$. Table B.5 on page 860 for modified Bessel functions:
$I_0(x), I_1(x), K_0(x), K_1(x)$ for $0 \leq x \leq 10$. The first five roots of characteristic equation for plane wall $\delta_n \sin(\delta_n) = Bi \cos(\delta_n)$ for a range of values of the Biot number. See Maple worksheets on ME 353 Web site how to calculate Bessel functions, and to obtain the roots of the characteristic equations for plane wall, long circular cylinder and solid sphere.

Lecture 2

Multidimensional systems: (i) finite circular cylinder with side cooling and end cooling, (ii) rectangular plate, and (iii) cuboid (parallelopiped). See Section 5.8.

• Finite circular cylinder of diameter D and length $2L$. Temperature excess depends on r, z, t. There are two sets of Biot and Fourier numbers: $Bi_1 = h_1 L/k, Fo_1 = \alpha t/L^2$ and $Bi_2 = h_2(D/2)/k, Fo_1 = \alpha t/(D/2)^2$. Here the subscripts $1, 2$ denote cooling at two ends and side respectively.
Dimensionless temperature excess is obtained from the plane wall and long circular cylinder solutions:

$$\left(\frac{\theta}{\theta_i}\right)_{cp} = \left(\frac{\theta}{\theta_i}\right)_c \times \left(\frac{\theta}{\theta_i}\right)_p$$

and heat loss fraction is obtained from

$$\left(\frac{Q}{Q_i}\right)_{cp} = \left(\frac{Q}{Q_i}\right)_c + \left(\frac{Q}{Q_i}\right)_p - \left(\frac{Q}{Q_i}\right)_c \times \left(\frac{Q}{Q_i}\right)_p$$

and $Q_i = \rho c_p \pi (D/2)^2 2L\, \theta_i$.

• Rectangular plate of dimensions: $2L_1, 2L_2, 2L_3$ corresponding to the x, y, z-coordinates respectively. The dimension $2L_3 >> 2L_1$ and $2L_2$. Heat transfer coefficients are h_1 on the L_2L_3-surfaces and h_2 on the L_1L_3-surfaces. The surfaces perpendicular to the $z-$coordinate are adiabatic. The two sets of Biot and Fourier numbers are: $Bi_1 = h_1L_1/k, Fo_1 = \alpha t/L_1^2$ and $Bi_2 = h_2L_2/k, Fo_2 = \alpha t/L_2^2$.
Dimensionless temperature excess which depends on (x, y, t) is obtained from the plane wall solution applied twice:

$$\left(\frac{\theta}{\theta_i}\right)_{p_1p_2} = \left(\frac{\theta}{\theta_i}\right)_{p_1} \times \left(\frac{\theta}{\theta_i}\right)_{p_2}$$

and heat loss fraction is obtained from

$$\left(\frac{Q}{Q_i}\right)_{p_1p_2} = \left(\frac{Q}{Q_i}\right)_{p_1} + \left(\frac{Q}{Q_i}\right)_{p_2} - \left(\frac{Q}{Q_i}\right)_{p_1} \times \left(\frac{Q}{Q_i}\right)_{p_2}$$

and $Q_i = \rho c_p 2L_1 2L_2 2L_3 \theta_i$.
See example 5.6 transient conduction in finite cylinder and Maple worksheets on ME 353 Web site.

• Cuboid of dimensions: $2L_1, 2L_2, 2L_3$ corresponding to the x, y, z-coordinates respectively. Heat transfer coefficients are h_1 on the L_2L_3-surfaces, h_2 on the L_1L_3-surfaces, and h_3 on the L_1L_2-surfaces. The three sets of Biot and Fourier numbers are: $Bi_1 = h_1L_1/k, Fo_1 = \alpha t/L_1^2$, $Bi_2 = h_2L_2/k, Fo_2 = \alpha t/L_2^2$ and $Bi_3 = h_3L_3/k, Fo_3 = \alpha t/L_3^2$.
Dimensionless temperature excess which depends on (x, y, z, t) is obtained from the plane wall solution applied three times:

$$\left(\frac{\theta}{\theta_i}\right)_{p_1p_2p_3} = \left(\frac{\theta}{\theta_i}\right)_{p_1} \times \left(\frac{\theta}{\theta_i}\right)_{p_2} \times \left(\frac{\theta}{\theta_i}\right)_{p_3}$$

and heat loss fraction is obtained from

$$\left(\frac{Q}{Q_i}\right)_{p_1p_2p_3} = \left(\frac{Q}{Q_i}\right)_{p_1} + \left(\frac{Q}{Q_i}\right)_{p_2} \times \left[1 - \left(\frac{Q}{Q_i}\right)_{p_1}\right] + \left(\frac{Q}{Q_i}\right)_{p_3} \times \left[1 - \left(\frac{Q}{Q_i}\right)_{p_1}\right] \left[1 - \left(\frac{Q}{Q_i}\right)_{p_2}\right]$$

and $Q_i = \rho c_p 2L_1 2L_2 2L_3 \theta_i$.

• Approximations of Bessel functions $J_0(x), J_1(x)$ based on trapezoidal rule applied to integral forms of the Bessel functions. Also see B.4, page 859 for short tables of $J_0(x), J_1(x)$ for $0 \leq x \leq 2.4$. The following relations are based on 4 panels:

$$J_0(x) = \frac{1}{4} + \frac{1}{2}\cos\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{4}\cos(x)$$

and

$$J_1(x) = \frac{1}{4}\sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{4}\right) + \frac{1}{4}\sin(x) - \frac{1}{4}\cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{4}\right)$$

These approximations can be used in the single term approximate solutions for long cylinders.

Lecture 3

Convective Heat Transfer.

The four chapters which deal with this topic are:

- Chapter 6: Review of fluid mechanics and definitions.
- Chapter 7: External forced convection.
- Chapter 7: Internal forced convection.
- Chapter 8: External and internal natural convection.

Convective heat transfer analysis is complex because, in general, one must solve 6 equations simultaneously to find the three velocity components: (u, v, w), the temperature T and the pressure: P.

The six equations are:

Continuity equation (1)

Momentum equations (3)

Energy equation (1)

Equation of state (1)

Correlation equations are based on (i) approximate analytical solutions, (ii) experiments and (iii) numerical solutions. The correlation equations relate the Nusselt number to other dimensionless groups.

For forced convection:

- Reynolds number
- Prandtl number
- Peclet number

for natural convection:

- Grashof number
- Rayleigh number

for internal forced convection:

- Stanton number
- Graetz number

See Table 6.2 on page 320 for a list of these and other dimensionless groups which appear in conduction, fluid mechanics, convective heat transfer, and mass transfer with convection.

Lecture 4

• Forced, laminar flow over a smooth, flat, isothermal plate of large extent.
• Fluid velocity and temperature at $x = 0, y > 0$ are uniform: U_∞, T_∞.
• Plate temperature is $T_w(T_s, T_0) > T_\infty$.
• Boundary layers are formed: hydrodynamic $\delta(x)$ and thermal $\Delta(x)$. Relative size depends on the Prandtl number. See Summary of Convective Heat Transfer Relations.
• Definitions of local heat transfer coefficient arise from application of Newton's Law of Cooling: $q = h(x)\,(T_w - T_\infty)$ and Fourier's Law of Conduction: $q = -k_f \partial T/\partial y$ at the wall $y = 0$ to obtain:

$$h(x) = \frac{-k_f\,(\partial T/\partial y)_{y=0}}{T_w - T_\infty}$$

• Local and area average heat transfer coefficients: $h(x)$ or h_x and $\bar{h} = (1/L)\int_0^L h(x)dx$.
