ME 353 M.M. Yovanovich

Week 4

Lecture 1

Solutions of one-dimensional Poisson equation in plane wall, long solid circular cylinder and solid sphere; all systems are convectively cooled;
see ME 353 Web Site for general equation and general solution with parameter n where $n = 0$ for plane wall, $n = 1$ for cylinder, and $n = 2$ for sphere;
surface temperature rise: $\boxed{T_s - T_f = \mathcal{P}b/((n+1)h)}$;
solid temperature rise: $\boxed{T_{max} - T_s = \mathcal{P}b^2/(2(n+1)k)}$;
Biot number for all cases: $\boxed{Bi = hb/(2k)}$;
$\boxed{\Delta T_{\text{solid}}/\Delta T_{\text{film}} = Bi/2}$ for all cases;
examples: nuclear fuel element with cladding; ohmic heating in wire with insulation.

Lecture 2

Hand out Project 1. Due Wednesday, October 8, 11:30 AM.
Extended surfaces or fins; see Web Site for derivation of general fin equation and general solution for fins with contact conductance, h_c, and end cooling, h_e; solution $\theta(x) = T(x) - T_f$ is a function of $Bi_c = h_c L/k, Bi_e = h_e L/k, mL$ with fin parameter: $mL = \sqrt{(hP/kA)}\,L$; P is the constant fin perimeter and A is the constant fin conduction area;
fin resistance: $\boxed{R_{\text{fin}} = \theta_b/Q}$; $\theta_b = T_b - T_f$; special cases of the general solution: (a) perfect contact: $Bi_c = \infty$ and end cooling; (b) perfect contact and adiabatic end: $Bi_e = 0$; (c) perfect contact and *infinitely* long fin; criterion for *long* fin: $L \geq 2.65/\sqrt{(hP/kA)}$;

Lecture 3

Show examples of pin fins, straight fins and circular annular fins from telecommunication and microelectronics industries, and automotive industries;
Sketch the typical straight fin (straight or pin) of length, L, conduction area, A, and perimeter, P. Contact conductance, h_c at the base, $x = 0$, and convective cooling at the end, $x = L$; when $Bi = ht_e/k < 0.2$, then $T(x, y) \to T(x)$; make

heat balance on differential control volume (CV): conduction into CV at x is $dQ_x = -kAdT/dx$, conduction rate out of CV at $x + dx$ is $Q_x + (dQ_x/dx)\,dx$, convection loss is $dQ_{\text{conv}} = hPdx(T(x) - T_f)$; no sources, steady-state; derive the governing ordinary differential equation; see section 3.6.2 for derivation of ODE and boundary conditions at $x = 0$ (perfect contact) and $x = L$ (convection cooling); see Table 3.4 for summary of the solutions.

Lecture 4

Makeup lecture 2;

Lectures on Monday at 11:30 AM and Tuesday at 12:30 AM in Professor Lenards lecture slots for the cancelled November lectures;

Review of material on Web site for derivation of general fin equation valid for variable conduction area, $A(x)$, and variable perimeter, $P(x)$; introduce temperature excess: $\theta(x) = T(x) - T_f$; note that $d\theta/dx = dT/dx$ because T_f is constant; consider special case: A and P are constants;

fin equation becomes $\boxed{d^2\theta/dx^2 - m^2\theta = 0}$ in $\boxed{0 \le x \le L}$ with fin parameter: $\boxed{m = \sqrt{hP/kA}}$ units of m are $1/m$;

at the base, $x = 0$, there is contact conductance, h_c, and at the fin end, $x = L$, there is convective cooling, h_e;

boundary conditions of the third kind (Robin) are applied at the fin base and fin end:

$\boxed{d\theta(0)/dx = -(h_c/k)\,[\theta_b - \theta(0)]}$ and $\boxed{d\theta(L)/dx = -(h_e/k)\theta(L)}$; $\theta_b = T_b - T_f$;

solution is $\boxed{\theta = C_1 \cosh mx + C_2 \sinh mx}$;

introduce dimensionless fin parameters: $\boxed{Bi_c = h_c L/k}$, $\boxed{Bi_e = h_e L/k}$, $\boxed{mL = \sqrt{(hP/kA)}\,L}$; $\boxed{Bi = ht_e/k < 0.2}$ where the effective fin thickness is defined as $\boxed{t_e = A/P}$;

solve for the constants of integration; obtain fin heat flow rate: $\boxed{Q_{\text{fin}} = -kA_b d\theta(0)/dx}$; A_b is the base conduction area; fin resistance: $\boxed{R_{\text{fin}} = \theta_b/Q_{\text{fin}}}$; see Web Site for development of solution and use the Javascript Calculator; fin efficiency: $\boxed{\eta_f = Q_{\text{fin}}/Q_{\text{ideal}}}$; Q_{ideal} corresponds to an ideal fin whose thermal conductivity is infinitely large;

special cases of general solution: perfect contact at fin base: $h_c = \infty$ and $Bi_c = \infty$; three options at the fin end: (i) perfect contact with fluid: $h_e = \infty$ and $Bi_e = \infty$; (ii) adiabatic end: $h_e = 0$ and $Bi_e = 0$; (iii) *infinitely* long fin,

i.e. $L > 2.65/\sqrt{hP/kA}$; see Web Site for several special cases.
Longitudinal fins; pin fins; circular annular fins; analytical solutions for several types of fins; see Table 3.5.