Week 2: Lecture 2

Isentropic Processes for Ideal Gases

Assume:

- constant specific heats over a wide range of temperature
- $ds = 0$
- $du = c_v dT \quad \equiv c_v = \left(\frac{\partial u}{\partial T}\right)_v$
- $dh = c_p dT \quad \equiv c_p = \left(\frac{\partial h}{\partial T}\right)_P$

For isentropic processes

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

where

$$k = \frac{c_p}{c_v}$$

Polytropic Processes for Ideal Gases

The isentropic process is a special case of a more general process known as a polytropic process, where

$$Pv^n = constant$$

isothermal process	$n = 1$	$Pv = RT = constant$
isobaric process	$n = 0$	$Pv^0 = constant$
isentropic process	$n = k$	$k = c_p/c_v$
constant volume process	$n \to \infty$	$Pv^\infty = constant$

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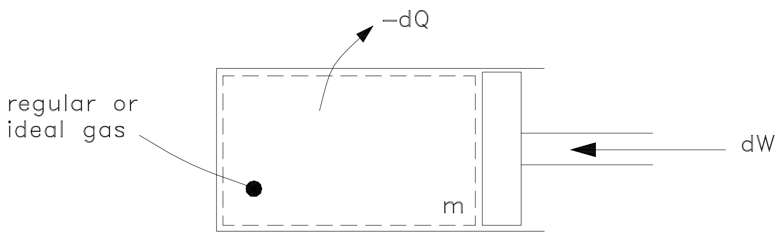
Reversible Compression/Expansion for a Fixed Mass:



From the first law

$$mdu = dW - dQ \quad (1)$$

From the second law

$$mds = -\frac{dQ}{T} \quad (2)$$

Combining (1) and (2) through the dQ term

$$mdu = dW + mTds \quad (3)$$

From Gibb's equation

$$mdu = mTds - mPdv \quad (4)$$

Combining (3) and (4)

$$\frac{W_{1-2}}{m} = \int_1^2 -Pdv$$

This is suitable for reversible processes with any gas, real or ideal.

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Summary

For a perfect gas with constant specific heat

$$
\begin{aligned}
Pv &= RT \\
u_2 - u_1 &= c_v (T_2 - T_1) \\
h_2 - h_1 &= c_p (T_2 - T_1) \\
s_2 - s_1 &= c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \\
&= c_p \ln \frac{T_2}{T_1} + R \ln \frac{P_2}{P_1} \\
&= c_p \ln \frac{v_2}{v_1} + R \ln \frac{P_2}{P_1} \\
R &= c_p - c_v
\end{aligned}
$$

For a reversible, isothermal process for an ideal gas

$$
\begin{aligned}
W_{1-2} &= mRT \ln \frac{V_1}{V_2} \\
&= mRT \ln \frac{P_1}{P_2}
\end{aligned}
$$