ME 354 THERMODYNAMICS - 2

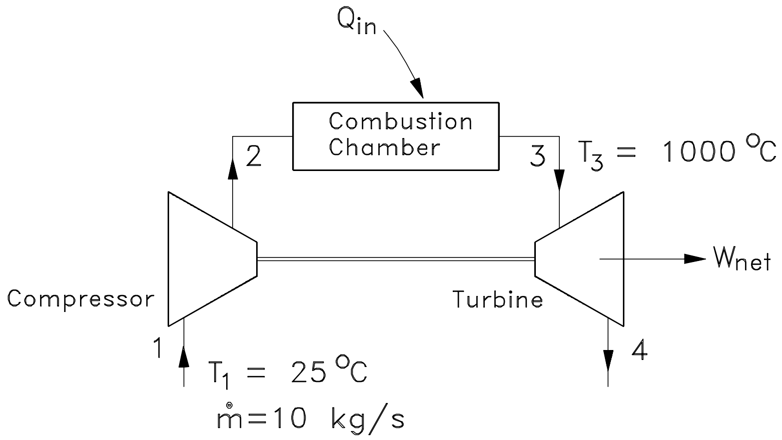
17 April 1998 Final Examination R. Culham

- This is a three-hour, closed-book examination.
- You are permitted to use one 8.5 in.× 11 in. crib sheet. (one side only)
- There are 4 questions to be answered. Read the questions very carefully.
- Clearly state all assumptions.
- It is your responsibility to write clearly and legibly.
- Good luck.

Question 1 (25 marks)

A gas turbine engine, as shown below, uses air as the working fluid with a constant specific heat of $c_p = 1.004\ kJ/(kg \cdot K)$ and $k = 1.4$. The compressor has an isentropic efficiency of 100% and a pressure ratio of 18:1, while the turbine has an isentropic efficiency of 85% and a pressure ratio of 18:1. Find the following:

a) the net power output (kW)
b) the overall thermal efficiency of the engine
c) the thermal efficiency of an equivalent ideal cycle
d) the percentage increase in power if pressure ratio was optimized for maximum work output
e) why is it difficult for a gaseous cycle to have Carnot efficiency?



Using the isentropic relationships, T_{2s} can be calculated as

$$T_{2s} = T_1 \times (r_p)^{k-1/k} = (25 + 273) \times (18)^{0.4/1.4} = 680.56\ K$$

Since the compressor efficiency is 100%, T_2 is the same as T_{2s}

$$T_2 = T_{2s} = 680.56\ K$$

Using the isentropic relationship for the turbine

$$T_{4s} = T_3 \times \left(\frac{1}{r_p}\right)^{k-1/k} = (1000 + 273) \times \left(\frac{1}{18}\right)^{0.4/1.4} = 557.42\ K$$

Since the mass flow rate and the specific heat are constant, the isentropic efficiency of the turbine can be written as

$$\begin{aligned} \eta_T &= \frac{T_3 - T_4}{T_3 - T_{4s}} \\ 0.85 &= \frac{1273 - T_4}{1273 - 557.42} \end{aligned}$$

Therefore

$$T_4 = 1273 - 0.85(1273 - 557.42) = 664.76\ K$$

The net work output per unit mass is

$$\begin{aligned} \frac{W_{net}}{m} &= c_p(T_3 - T_4) - c_p(T_2 - T_1) \\ &= 1.004\ kJ/kg \cdot K\ [(1273 - 664.76) - (680.56 - 298)]\ K \\ &= 226.58\ kJ/kg \end{aligned}$$

Part a)
The power output is

$$Power = \dot{m} \times \frac{W_{net}}{m} = 10\ kg/s\ \times\ 226.58\ kJ/kg = 2265.8\ kW$$

Part b)
The thermal efficiency of the engine is

$$\begin{aligned} \eta &= \frac{W_{net}}{Q_{in}} = \frac{T_3 - T_4 - T_2 + T_1}{T_3 - T_2} \\ &= \frac{1273 - 664.76 - 680.56 + 298}{1273 - 680.56} \\ &= 0.381 = 38.1\% \end{aligned}$$

Note: $\eta = 1 - (r_p)^{1-k/k}$ can only be used for the ideal Brayton cycle where $T_4 = T_{4s}$

Part c)

The thermal efficiency for an ideal Brayton cycle is given by

$$\begin{aligned}\eta &= 1 - (r_p)^{(1-k)/k} \\ &= 1 - (18)^{(1-1.4)/1.4} \\ &= 0.5621\end{aligned}$$

Part d)

The optimum pressure ratio is

$$r_{p,opt} = \left(\frac{T_3}{T_1}\right)^{k/(2k-2)} = \left(\frac{1273}{298}\right)^{1.4/.8} = 12.69$$

$$T_2 = T_{2s} = T_1 \times (r_p)^{k-1/k} = (25 + 273) \times (12.69)^{0.4/1.4} = 615.87\ K$$

$$T_{4s} = T_3 \times \left(\frac{1}{r_p}\right)^{k-1/k} = (1000 + 273) \times \left(\frac{1}{12.69}\right)^{0.4/1.4} = 615.96\ K$$

$$T_4 = 1273 - 0.85(1273 - 615.96) = 714.52\ K$$

$$\begin{aligned}\frac{W_{net}}{m} &= c_p(T_3 - T_4) - c_p(T_2 - T_1) \\ &= 1.004\ kJ/kg \cdot K\ [(1273 - 714.52) - (615.87 - 298)]\ K \\ &= 241.57\ kJ/kg\end{aligned}$$

The power output is

$$Power = \dot{m} \times \frac{W_{net}}{m} = 10\ kg/s\ \times\ 241.57\ kJ/kg = 2415.7\ kW$$

The percentage increase in power is

$$Percent\ change = \frac{2415.7 - 2265.8}{2265.8} \times 100 = 6.61\%$$

Part e)

It is difficult to keep the temperature constant while adding or subtracting energy by heat. This implies changes of pressure and specific volume.

Question 2 (25 marks)

A small gas turbine uses $C_8H_{18}(\ell)$ for fuel and 400% theoretical air. The air and fuel enter at 25 $°C$ and 1 atm, and the products of combustion leave at 900 K. The output of the engine and the fuel consumption are measured and it is found that the specific fuel consumption is 0.25 kg/s of fuel per megawatt of output. Assume a steady-state, steady flow process, all gases are ideal and the combustion is complete.

a) determine the heat transfer from the engine per kmole of fuel

b) if we assume the turbine is adiabatic and the work output is negligible show that the maximum temperature of the products is approximately 960 K. Why is this temperature so low?

Part a)

The combustion equation is

$$C_8H_{18}(\ell) + 4(12.5)O_2 + 4(12.5)(3.76)N_2 \rightarrow 8CO_2 + 9H_2O + 37.5O_2 + 188.0N_2$$

From a first law energy balance on the system we obtain

$$Q_{CV} + \sum_R n_i(\bar{h}_f^o + \Delta\bar{h})_i = W_{CV} + \sum_P n_e(\bar{h}_f^o + \Delta\bar{h})_e$$

Since the air is composed of elements and is at STP, the enthalpy of the reactants is equal to that of the fuel,

$$\sum_R n_i(\bar{h}_f^o + \Delta\bar{h})_i = (\bar{h}_f^o)_{C_8H_{18}(\ell)} = -250,105 \; kJ/kmole \; fuel$$

Considering the products we have

$$\begin{aligned}
\sum_P n_e(\bar{h}_f^o + \Delta\bar{h})_e &= n_{CO_2}((\bar{h}_f^o + \Delta\bar{h})_{CO_2} + n_{H_2O}((\bar{h}_f^o + \Delta\bar{h})_{H_2)} \\
&\quad + n_{O_2}(\bar{h}_f^o + \Delta\bar{h})_{O_2} + n_{N_2}((\bar{h}_f^o + \Delta\bar{h})_{N_2} \\
&= 8(-393,522 + 28,030) + 9(-241,826 + 21,892) \\
&\quad + 37.5(0 + 19,249) + 188(0 + 18,222) \\
&= -755,769 \; kJ/kmole \; fuel
\end{aligned}$$

The work can be calculated as

$$W_{CV} = \frac{1000 \; kJ/s}{0.25 \; kg/s} \times \frac{114.23 \; kg}{kmole} = 456,920 \; kJ/kmole \; fuel$$

Therefore from the first law

$$\begin{aligned}
Q_{CV} &= -755,769 + 456,920 - (-250,105) \\
&= -48,744 \; kJ/kmolefuel
\end{aligned}$$

Part b)

We know that for an adiabatic system with no work output

$$\sum_R n_i(\bar{h}_f^o + \Delta\bar{h})_i = \sum_P n_e(\bar{h}_f^o + \Delta\bar{h})_e$$

which can be rearranged as

$$\underbrace{\sum_P n_e \Delta\bar{h}_e}_{LHS} = \underbrace{\sum_R n_i(\bar{h}_f^o + \Delta\bar{h})_i - \sum_P n_e \bar{h}_f^o}_{RHS}$$

The RHS is known and has a value of

$$\begin{aligned} RHS &= -250,105\ kJ/kmole\ fuel - (8\ \times\ -393,522 + 9\ \times\ -241,826 + 0 + 0)\ kJ/kmole\ fuel \\ &= 5,074,505\ kJ/kmole\ fuel \end{aligned}$$

The LHS at 960 K is

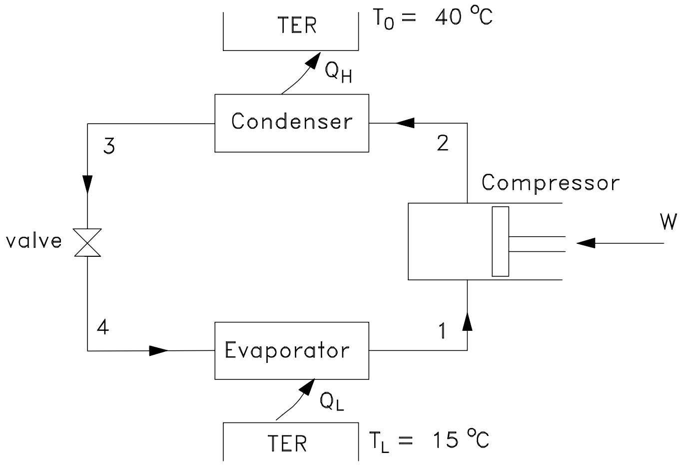
$$\begin{aligned} LHS &= 8\ \times (31,250) + 9\ \times\ (24,375) + 37.5\ \times\ (21318) + 188\ \times\ (20,167) \\ &= 5,060,196\ kJ/kmole\ fuel \end{aligned}$$

QED (close enough)

Question 3 (25 marks)

A computer facility in the Sahara Desert [$T_0 = 40°C$] is to be maintained at $15°C$ by a vapour-compression refrigeration system that uses water as the refrigerant. The water leaves the evaporator as a saturated vapour at $10°C$. The compressor is reversible and adiabatic. The pressure in the condenser is 0.01 MPa and the water is saturated liquid as it leaves the condenser.

a) determine the coefficient of performance for the cycle
b) determine the second law efficiency of the system, defined as the availability per unit work input
c) if we assume that the valve is heavily insulated, determine the total availability destruction per unit mass of water for the system
d) what caused the exergy destruction in this particular process?



State	T ($°C$)	P (MPa)	h (kJ/kg)	s ($kJ/kg \cdot K$)
1	10	0.0012276	2519.8	8.9008
2	$\approx$ 200	0.01	2878.2	8.9008
3	45.29	0.01	191.6	.6483
4	10	0.0012276	191.6	.6792

$x_4 = 0.06037$

Part a)

$$COP = \frac{q_L}{w_{in}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{2519.8 - 191.6}{2878.2 - 2519.8} = 6.5$$

Part b)
The change in availability across the evaporator is given as

$$\psi_e - \psi_i = (h_e - h_i) - T_0(s_e - s_i)$$

$$
\begin{aligned}
\eta_{2nd} &= \frac{|\text{availability}|}{\text{net work output}} \\
&= |(h_1 - h_4) - T_0(s_1 - s_4)|/(h_2 - h_1) \\
&= |(2519.8 - 191.6) - 313(8.9008 - .6792)/(2878.2 - 2519.8) \\
&= 0.684
\end{aligned}
$$

Part c)

From the second law, the entropy production of our system is given by

$$
\mathcal{P}_s = \left(\frac{q_H}{T_0} - \frac{q_L}{T_L}\right)
$$

The total availability destruction per unit mass is given as

$$
\begin{aligned}
i_{tot} &= T_0 \mathcal{P}_s \\
&= T_0 \left[\frac{q_H}{T_0} - \frac{q_L}{T_L}\right] \\
&= q_H - q_L \frac{T_0}{T_L} \\
&= (h_2 - h_3) - (h_1 - h_4)\left(\frac{40 + 273}{15 + 273}\right) \\
&= 156.3\ kJ/kg
\end{aligned}
$$

As a check, we know

$$
\begin{aligned}
i &= T_0 \left(s_e - s_i - \frac{q_k}{T_k}\right) \\
\psi_e - \psi_i &= (h_e - h_i) - T_0(s_e - s_i)
\end{aligned}
$$

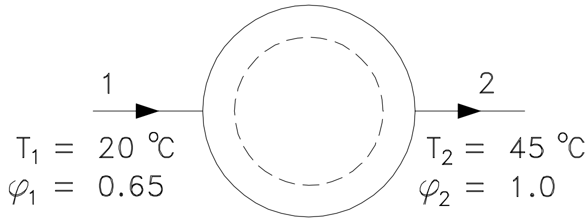
	q	w	Δh	Δs	i	$\Delta\psi$
1-2	0	-358.4	358.4	0	0	358.4
2-3	-2686.6	0	-2686.6	-8.253	103.41	-103.41
3-4	0	0	0	0.0309	9.67	-9.67
4-1	2328.2	0	2328.2	8.22	42.56	-244.66
Total					155.64	

Part d)

The availability destruction represents the opportunity to cool the computer facility that is irrevocably lost in a heat transfer process across a finite temperature difference and due to viscous fluid flow through the valve.

<u>Question 4</u> (25 marks)

Air enters a tumble dryer at 20 $°C$ and a relative humidity of 65%. If the air leaves the dryer in a saturated condition at 45 $°C$, calculate the mass flow rate of dry air through the dryer in order to evaporate 1 kg of water in 40 minutes. Assume the process to take place at a constant atmospheric pressure of 100 kPa.



The saturation pressure can be obtained from the table for saturated water.

$$P_{sat_1} = 2.339 \ kPa$$

The relative humidity at state point 1 can be written as

$$\phi_1 = 0.65 = \frac{P_{w_1}}{P_{sat_1}} = \frac{P_{w_1}}{2.339}$$

Therefore

$$P_{w_1} = 1.520 \ kPa$$

We also know that

$$\gamma = 0.622 \ \frac{P_w}{P_a}$$

and

$$P_{a_1} = P - P_{w_1} = 100 - 1.520 = 98.48 \ kPa$$

Therefore

$$\gamma_1 = 0.622 \ \frac{1.520}{98.48} = 0.0096 \ kg/(kg \ air)$$

At state point 2, $T_2 = 45 \ °C$ and $\phi_2 = 1.0$. From the saturated water tables

$$P_{sat_2} = 9.593 \ kPa$$

Now

$$P_{a_2} = P - P_{w_2} = 100 - 9.593 = 90.407 \ kPa$$

and

$$\gamma_2 = 0.622 \ \frac{9.593}{90.407} = 0.066 \ kg/(kg \ air)$$

Therefore the amount of water evaporated is

$$\dot{m}_{w_2} - \dot{m}_{w_1} = \frac{1}{40 \ \times \ 60} = 0.000417 \ kg/s$$

By definition

$$\gamma_2 - \gamma_1 = \frac{\dot{m}_{w_2}}{\dot{m}_a} - \frac{\dot{m}_{w_1}}{\dot{m}_a}$$

Therefore

$$\begin{aligned}
\dot{m}_a &= \frac{\dot{m}_{w_2} - \dot{m}_{w_1}}{\gamma_2 - \gamma_1} \\
&= 0.000417/(0.066 - 0.0096) \\
&= 0.00739 \ kg/s \\
&= 26.62 \ kg/hr
\end{aligned}$$