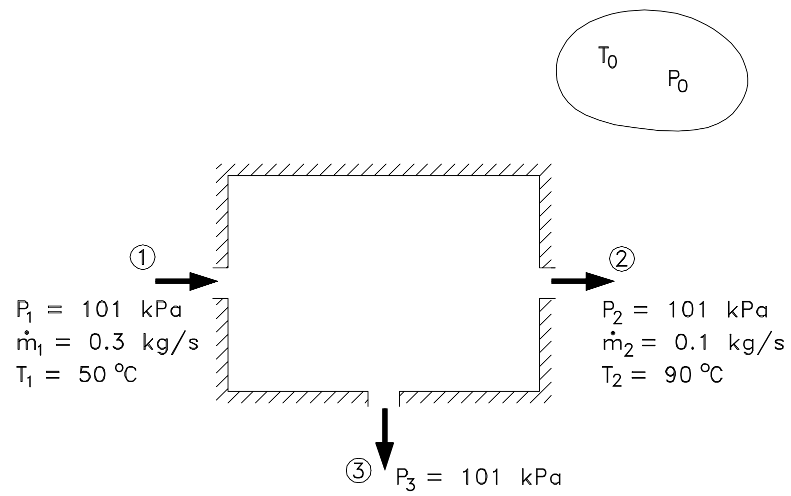
ME 354 THERMODYNAMICS - 2

Midterm Solution Set
Spring '98

Problem 1:

Air enters a black box at 50 $°C$, 101 kPa at 0.3 kg/s. The air flows steadily through the box and leaves through two different exits. At one exit the air leaves at 90 $°C$, 101 kPa at a rate of 0.1 kg/s. The air pressure is 101 kPa at the other exit. There is no work or heat transfer entering or leaving the box. Assume that the specific heat of the air is constant at 1004 $J/(kg \cdot K)$.

a) Determine the unknown exit temperature.
b) Determine the rate of availability destruction within the black box if $T_0 = 25$ $°C$ and $P_0 = 101$ kPa.
c) In this particular example, is it possible for the destruction term to be negative? Why or why not?



Solution

Part a)

Conservation of mass over the black box gives:

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

Solving for $\dot{m}_3$

$$\begin{aligned} \dot{m}_3 &= \dot{m}_1 - \dot{m}_2 \\ &= 0.3 - 0.1 \\ &= 0.2 \ kg/s \end{aligned}$$

Conservation of energy over the black box yields:

$$\begin{aligned}\dot{m}_1 h_1 &= \dot{m}_2 h_2 + \dot{m}_3 h_3 \\ &= \dot{m}_2 h_2 + (\dot{m}_1 - \dot{m}_2) h_3\end{aligned}$$

Collecting common terms

$$\begin{aligned}\dot{m}_1 (h_1 - h_3) &= \dot{m}_2 (h_2 - h_3) \\ \dot{m}_1 c_p (T_1 - T_3) &= \dot{m}_2 c_p (T_2 - T_3)\end{aligned}$$

or

$$\begin{aligned}T_3 &= \frac{\dot{m}_1 T_1 - \dot{m}_2 T_2}{\dot{m}_1 - \dot{m}_2} \\ &= \frac{0.3\ kg/s\ (273 + 50)\ K\ -\ 0.1\ kg/s\ (273 + 90)\ K}{0.2\ kg/s} \\ &= 303\ K\ =\ 30\ ^\circ C\end{aligned}$$

Part b)

From the second law we know that

$$\dot{m}_1 s_1 + \dot{\mathcal{P}}_s = \dot{m}_2 s_2 + \dot{m}_3 s_3$$

Solving for the entropy production term

$$\begin{aligned}\dot{\mathcal{P}}_s &= \dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1 \\ &= \dot{m}_2 s_2 + (\dot{m}_1 - \dot{m}_2) s_3 - \dot{m}_1 s_1 \\ &= \dot{m}_1 (s_3 - s_1) + \dot{m}_2 (s_2 - s_3)\end{aligned}$$

The rate of availability destruction is defined as

$$\begin{aligned}\dot{I} = T_0 \dot{\mathcal{P}}_s &= T_0 [\dot{m}_1 (s_3 - s_1) + \dot{m}_2 (s_2 - s_3)] \\ &= T_0 \left[\dot{m}_1 c_p \ln\left(\frac{T_3}{T_1}\right) + \dot{m}_2 c_p \ln\left(\frac{T_2}{T_3}\right)\right] \\ &= c_p T_0 \left[\dot{m}_1 \ln\left(\frac{T_3}{T_1}\right) + \dot{m}_2 \ln\left(\frac{T_2}{T_3}\right)\right] \\ &= 1004\ J/(kgK) \times 298\ K\ [0.3\ kg/s\ \ln\left(\frac{303\ K}{323\ K}\right)\ + 0.1\ kg/s\ \ln\left(\frac{363\ K}{303\ K}\right) \\ &= -331.76\ J/s = -0.33\ kW\end{aligned}$$

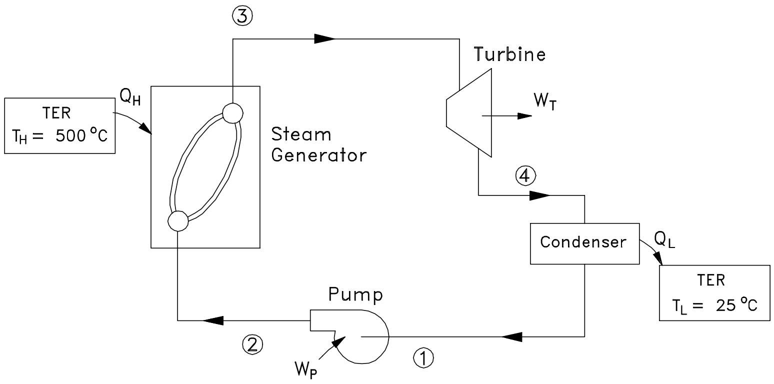
Part c)

As described, the system (i.e. the black box) and the surroundings are not isolated. The entropy production of the black box can be negative but the overall entropy production of the system and the surroundings in an isolated system and in turn the availability destruction will always be positive.

Problem 2:

A Rankine-cycle, steam power plant operates between a source temperature at 500 $°C$ and the reference atmosphere at 25 $°C$. For safety reasons the boiler pressure cannot be more than 10 MPa. The mass-flow rate of steam is 50 kg/s. The pumping process is in the liquid region. Assume that both the pump and the turbine are isentropic.

a) Determine the maximum first-law efficiency.
b) Determine the maximum second-law efficiency.
c) Determine the electrical generating capacity [MW].



Solution

State	T ($°C$)	P (MPa)	ν (m^3/kg)	h (kJ/kg)	s (kJ/kgK)
1	25	.003169	0.001003	104.9	0.3672
2		10		114.9	0.3672
3	500	10		3373.6	6.5974
4		.003169		1985.5	6.5974

At the isentropic pump we know that:

$$Tds = dh - v \cdot dP = 0$$

Therefore

$$h_2 - h_1 = \nu(P_2 - P_1)$$

$$\begin{aligned} h_2 &= h_1 + \nu(P_2 - P_1) \\ &= 104.9 + 0.001003(10,000 - 3.169) \\ &= 114.9\ kJ/kg \end{aligned}$$

At state point 4 we know that:

$$s_4 = (1-x)s_f + x \cdot s_g$$

$$\begin{aligned} x &= (s - s_f)/s_{fg} = (6.5974 - 0.3672)/8.1916 \\ &= 0.77 \end{aligned}$$

and

$$\begin{aligned} h_4 &= (1-x)h_f + x \cdot h_g \\ &= (1-.77) \cdot 104.9 + 0.77 \cdot 2597.2 \\ &= 1985.5 \ kJ/kg \end{aligned}$$

<u>Part a)</u>

$$\begin{aligned} \frac{\dot{W}_{net}}{\dot{m}} &= (h_3 - h_4) - (h_2 - h_1) \\ &= (3373.6 - 1985.5) - (114.9 - 104.9) \\ &= 1378.1 \ kJ/kg \end{aligned}$$

$$\begin{aligned} \frac{\dot{Q}_B}{\dot{m}} &= h_3 - h_2 \\ &= 3373.6 - 114.9 \\ &= 3258.7 \ kJ/kg \end{aligned}$$

The first law efficiency is

$$\eta_{1st} = \frac{\dot{W}_{net}/\dot{m}}{\dot{Q}_B/\dot{m}} = \frac{1378.1}{3258.7} = 0.423$$

<u>Part b)</u>

The second law efficiency is

$$\begin{aligned} \eta_{2nd} &= \frac{net\ work\ output}{availability} \\ &= \frac{\dot{W}_{net}/\dot{m}}{\dfrac{\dot{Q}_B}{\dot{m}}\left[1 - \dfrac{T_0}{T_H}\right]} = \frac{\eta_{1st}}{\left[1 - \dfrac{T_0}{T_H}\right]} = \frac{0.423}{1 - \dfrac{298.2}{773.2}} = 0.689 \end{aligned}$$

<u>Part c)</u>

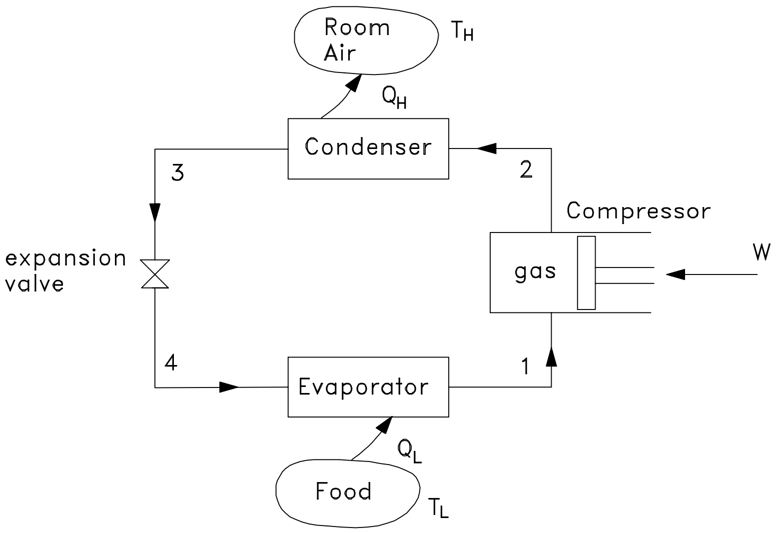
$$\dot{W}_{net} = \dot{m} \times \frac{\dot{W}_{net}}{\dot{m}}$$

$$
\begin{aligned}
&= 50\ kg/s \quad \times \quad 1378.1\ kJ/kg\ \frac{1\ MW - s}{10^3\ kJ} \\
&= 68.9\ MW
\end{aligned}
$$

Problem 3:

A vapour-compression refrigerator using R-12 has a maximum pressure of 800 kPa and a minimum pressure of 120 kPa. The R-12 is saturated liquid leaving the condenser and saturated vapour leaving the evaporator. The compressor is reversible and adiabatic.

a) Determine the maximum room temperature and the minimum food compartment temperature.
b) Determine the coefficient of performance.
c) Is the expansion valve isentropic? If not, what is the change in entropy?
d) Determine the availability destroyed in the condenser $[kJ/kg]$ if $T_0 = T_{room}$, where T_{room} is from part a)



Solution

Part a)

From the R-12 tables at $P = 0.8\ MPa$

$$T_{room} = 32.74\ °C$$

From the R-12 tables at $P = 0.12\ MPa$

$$T_{food} = -25.74\ °C$$

Part b)

State	T $(°C)$	P (MPa)	h (kJ/kg)	s (kJ/kgK)	
1		.12	176.14	0.7133	vapour
2		.8	209.61	0.7133	
3		.8	67.3	0.2487	liquid
4		.12	67.3	0.2734	

At state point 4 we know that:

$$h_4 = (1-x)h_f + x \cdot h_g$$

$$\begin{aligned} x &= (h - h_f)/h_{fg} = (67.3 - 12.66)/163.48 \\ &= 0.3342 \end{aligned}$$

and

$$\begin{aligned} s_4 &= (1-x)s_f + x \cdot s_g \\ &= (1 - .3342) \cdot (0.0526) + 0.3342 \cdot 0.7133 \\ &= 0.2734 \; kJ/kg \cdot K \end{aligned}$$

From the superheated tables for R-12 a value of $s_2 = 0.7133$ is obtained at a temperature of $T \approx 45\ ^{\circ}C$

$$h_2 = (205.924 + 213.290)/2 = 209.61 \; kJ/kg$$

The COP is given by:

$$\frac{h_1 - h_4}{h_2 - h_1} = \frac{176.14 - 67.3}{209.61 - 176.14} = 3.25$$

Part c)

The expansion is not isentropic. The change in entropy is

$$\begin{aligned} s_4 - s_3 &= .2734 - .2487 \\ &= .0247 \; kJ/kg \cdot K \end{aligned}$$

Part d)

The availability destroyed per unit mass in the condenser is given by

$$\frac{\dot{I}}{\dot{m}} = T_0 \frac{\dot{\mathcal{P}}_s}{\dot{m}}$$

where the entropy production can be obtained by combining the first and second laws
1st law:

$$\frac{\dot{Q}}{\dot{m}} = h_2 - h_3$$

2nd law:

$$\dot{m} s_2 - \dot{m} s_3 - \frac{\dot{Q}}{T_0} + \dot{\mathcal{P}}_s = 0$$

Combining the above two equations

$$\begin{aligned} T_0 \frac{\dot{\mathcal{P}}_s}{\dot{m}} &= (h_2 - h_3) - T_0(s_2 - s_3) \\ &= (209.61 - 67.3) - (32.74 + 273)(0.7133 - 0.2487) \\ &= 0.2632 \; kJ/kg \end{aligned}$$