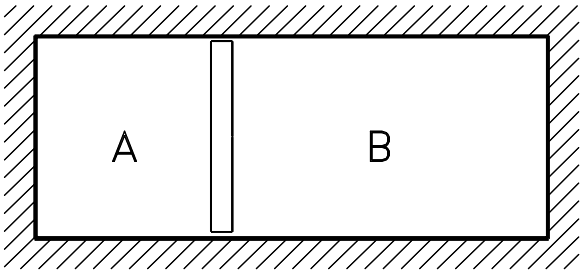
ME 354 THERMODYNAMICS - 2

Midterm Solution Set
Winter '98

Problem 1:

Consider a rigid, adiabatic tank with a movable piston as shown below. Initially, Side A contains 2.5 kg of air at 250 $°C$ and 500 kPa, while Side B contains 0.5 kg of air at 70 $°C$ and 50 kPa. The piston then moves (slowly because of friction) to a new position where the pressures on each side will be equal. Since the piston is not adiabatic, the temperatures on each side will eventually equalize.

a) Determine the final temperature.
b) Determine the final pressure.
c) Determine the availability destroyed $[kJ]$ if the dead state temperature and pressure are $T_0 = 20\ °C$ and $P_0 = 1\ atm$, respectively.



Solution

Given:

$$
\begin{aligned}
m_A &= 2.5\ kg & m_B &= 0.5\ kg \\
T_{A_1} &= 250\ °C & T_{B_1} &= 70\ °C \\
P_{A_1} &= 500\ kPa & P_{A_1} &= 50\ kPa
\end{aligned}
$$

Let subscript 1 denote the initial state and subscript 2 denote the final state.

a) Using a 1st law energy balance:

$$
\begin{aligned}
0 &= (U_{A_2} - U_{A_1}) + (U_{B_2} - U_{B_1}) \\
&= m_A \times c_v \times (T_{A_2} - T_{A_1}) + m_B \times c_v \times (T_{B_2} - T_{B_1})
\end{aligned}
$$

We know that $T_{A_2} = T_{B_2} = T_2$, therefore

$$
\begin{aligned}
T_2 &= \frac{m_A \times T_{A_1} + m_B \times T_{B_1}}{m_A + m_B} \\
&= \frac{2.5\ kg \times (250 + 273)K + 0.5\ kg \times (70 + 273)K}{(2.5 + 0.5)kg} \\
&= 493\ K = 220\ °C
\end{aligned}
$$

b) From the ideal gas equation

$$
\begin{aligned}
P_2 &= \frac{m_2 \times R \times T_2}{V_2} \\
&= \frac{m_2 \times R \times T_2}{V_{A_1} + V_{B_1}} \\
&= \frac{m_2 \times R \times T_2}{\left(\frac{m_A \times R \times T_{A_1}}{P_{A_1}}\right) + \left(\frac{m_B \times R \times T_{B_1}}{P_{B_1}}\right)} \\
&= \frac{(2.5 + 0.5)\ kg \times 493\ K}{\left(\frac{2.5\ kg \times 523\ K}{500\ kPa}\right) + \left(\frac{0.5\ kg \times 343\ K}{50\ kPa}\right)} \\
&= 244.7\ kPa
\end{aligned}
$$

c) From a 2nd law entropy balance for a control mass where for an adiabatic system, $dQ = 0$.

$$
\begin{aligned}
\mathcal{P}_s &= (S_{A_2} - S_{A_1}) + (S_{B_2} - S_{B_1}) \\
&= m_A \times (s_{A_2} - s_{A_1}) + m_B \times (s_{B_2} - s_{B_1})
\end{aligned}
$$

Availability destruction is give by

$$
I = T_0 \times \mathcal{P}_s = T_0 \times [m_A \times (s_{A_2} - s_{A_1}) + m_B \times (s_{B_2} - s_{B_1})]
$$

The change in entropy between state 1 and 2 can be obtained from the state equations for a perfect gas, where

$$
s_2 - s_1 = c_p \times \ln\left(\frac{T_2}{T_1}\right) - R \times \ln\left(\frac{P_2}{P_1}\right)
$$

From Table C-5b, c_p @ $493\ K = 1.029\ kJ/(kg \cdot K)$ and from Table B-6b, for air $R = 0.286\ kJ/(kg \cdot K)$.

$$
\begin{aligned}
s_{A_2} - s_{A_1} &= 1.029\ kJ/kg \cdot K \times \ln\left(\frac{493}{523}\right) - 0.286\ kJ/kg \cdot K \times \ln\left(\frac{244.7\ kPa}{500\ kPa}\right) \\
&= 0.1436\ kJ/(kg \cdot K)
\end{aligned}
$$

$$
\begin{aligned}
s_{B_2} - s_{B_1} &= 1.029\ J/kg \cdot K \times \ln\left(\frac{493}{343}\right) - 0.287\ kJ/kg \cdot K \times \ln\left(\frac{244.7\ kPa}{50\ kPa}\right) \\
&= -0.0809\ kJ/(kg \cdot K)
\end{aligned}
$$

$$
\begin{aligned}
I &= 293\ K \times [2.5\ kg \times 0.1436\ kJ/kg \cdot K + 0.5\ kg \times (-0.0809)\ kJ/kg \cdot K] \\
&= 93.34\ kJ
\end{aligned}
$$

Problem 2

Air enters an adiabatic gas turbine at 900 $°C$, 275 kPa and leaves at 110 kPa. The turbine efficiency is 0.80. The mass flow rate is 1000 kg/hr. Assume constant specific heat.

a) Determine the rate of work transfer $[kW]$
b) Determine the exit temperature.
c) Determine the rate of flow availability $[kW]$ leaving with the exit flow stream if $T_0 = 20\ °C$ and $P_0 = 101\ kPa$.

Solution

Part a)

From a first law energy balance

$$\dot{m} \cdot h_1 = \dot{W} + \dot{m} \cdot h_2$$

$$\dot{W} = \dot{m} \times (h_1 - h_2) = \dot{m} \times c_p \times (T_1 - T_2)$$

For the isentropic turbine

$$\dot{W}_s = \dot{m} \times (h_1 - h_{2s}) = \dot{m} \times c_p \times (T_1 - T_{2s})$$

We know that for an isentropic process with an ideal gas

$$\frac{T_{2s}}{T_1} = \frac{P_2}{P_1}^{(k-1)/k}$$

With $k = 1.4$ for air

$$T_{2s} = (900 + 273)\ K \times \frac{110\ kPa}{275\ kPa}^{(1.4-1)/1.4} = 902.82\ K = 629.82\ °C$$

From Table C-5b, the specific heat calculated at a mean turbine temperature of 1037.91 K is 1.149 $kJ/kg \cdot K$.

The isentropic work transfer rate is

$$\begin{aligned}
\dot{W}_s &= \dot{m} \times c_p \times (T_1 - T_{2s}) \\
&= 1000\ \frac{kg}{hr} \times 1.149\ \frac{kJ}{kg \cdot K} \times (1173 - 902.82)\ K \times \frac{1\ hr}{3600\ sec} \\
&= 86.23\ kW
\end{aligned}$$

Using an efficiency of 0.8 the work transfer rate is

$$\dot{W} = \eta \times \dot{W}_s = 0.8 \times 86.23\ kW = 68.98\ kW$$

Part b)

From our original energy balance

$$\begin{aligned}
T_2 &= T_1 - \frac{\dot{W}}{\dot{m} \times c_p} \\
&= 1173\ K - \frac{68.98\ kW \times \dfrac{kJ/s}{kW}}{1000 \dfrac{kg}{hr} \times \dfrac{1\ hr}{3600\ s} \times 1.149\ \dfrac{kJ}{kg \cdot K}} \\
&= 956.87\ K
\end{aligned}$$

Part c)
The flow availability is given as

$$\dot{m} \times \psi_2 = \dot{m}[(h_2 - h_0) - T_0(s_2 - s_1)]$$

From the equations of state for an ideal gas we know

$$\begin{aligned}
s_2 - s_0 &= c_p \times \ln\left(\frac{T_2}{T_0}\right) - R \times \ln\left(\frac{P_2}{P_0}\right) \\
&= 1.149\ \frac{kJ}{kg \cdot K} \times \ln\left(\frac{956.87\ K}{293\ K}\right) - 0.286\ \frac{kJ}{kg \cdot K} \times \ln\left(\frac{110\ kPa}{101\ kPa}\right) \\
&= 1.335\ \frac{kJ}{kg \cdot K}
\end{aligned}$$

We also know that

$$\begin{aligned}
h_2 - h_0 &= c_p(T_2 - T_0) \\
&= 1.149\ \frac{kJ}{kg \cdot K}(956.87\ K - 293\ K) \\
&= 762.79\ \frac{kJ}{kg}
\end{aligned}$$

Therefore the flow availability is

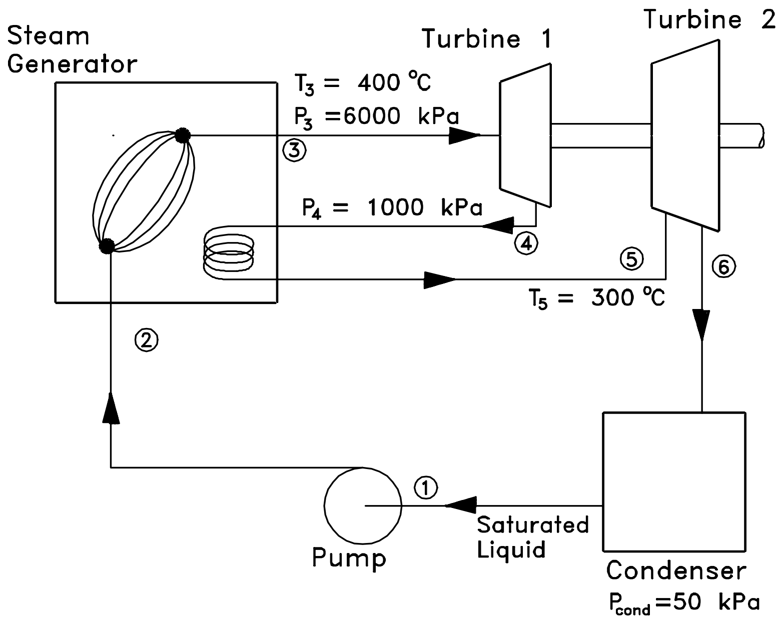
$$\begin{aligned}
\dot{m} \cdot \psi_2 &= 1000\ \frac{kg}{hr} \times \frac{1\ hr}{3600\ s} \times \left[762.79\ \frac{kJ}{kg} - 293\ K\left(1.335\ \frac{kJ}{kg \cdot K}\right)\right] \\
&= 103.23\ kW
\end{aligned}$$

Problem 3:

Due to a problem in copying the steam tables (Table B-2) two solutions are possible. i) using the tables as copied the values of enthalpy and entropy for $P_5 = 1\ MPa$ and $T_5 = 300\ °C$ are $h_5 = 3263.9\ kJ/kg$ and $s_5 = 7.4658\ kJ/kg \cdot K$. ii) using the actual tables from Reynolds and Perkins gives $h_5 = 3051.2\ kJ/kg$ and $s_5 = 7.1237\ kJ/kg \cdot K$. Both answers are acceptable and solution ii) will be included in the following solution with changes indicated in braketed text. i.e. $[\cdots]$

A reheat Rankine cycle that uses water as its working fluid operates a boiler at 6000 kPa, the reheat section at 1000 kPa, and the condenser at 50 kPa. The inlet temperature at turbine 1 is 400 $°C$ while the inlet temperature at turbine 2 is 300 $°C$. The system is sized such that the net power produced by the turbines is 300 kW. Assume all processes are steady state and reversible.

a) Determine the rate at which heat is added in the steam generator.
b) Determine the power required to operate the pump.
c) Determine the cycle's thermal efficiency.
d) Why is reheat used in a Rankine cycle?
e) Determine the cycle's thermal efficiency if reheat was not included in the process. Assume the same mass flow rate used in part a).



Solution

A summary of given data and data obtained from the attached tables is as follows:

State	P kPa	T $°C$	x $\%$	h kJ/kg	s $kJ/kg \cdot K$	v m^3/kg
1	50	80.9	0	338.7	1.08575	0.0010295
2	6000			344.83		$v_2 \approx v_1$
3	6000	400		3177.2	6.5415	
4	1000	179.9	98.97	2757.34	$s_4 = s_3$	
5	1000	300		3263.9	7.4658	
				[3051.2]	[7.1237]	
6	50	80.9	97.91	2596.90	$s_6 = s_5$	
			[92.66]	[2475.81]		

Part a)

Application of the first law to turbine 1 gives:

$$\frac{\dot{W}_{3-4}}{\dot{m}} = h_3 - h_4 = 3177.2 - 2757.34 = 419.86 \; kJ/kg$$

Similarly, applying the first law to turbine 2 gives:

$$\begin{aligned}\frac{\dot{W}_{5-6}}{\dot{m}} = h_5 - h_6 &= 3263.9 - 2596.90 = 667.3 \; kJ/kg \\ &= [3051.2] - [2475.81] = [575.39] \; kJ/kg\end{aligned}$$

The mass flow rate through the system is:

$$\begin{aligned}\dot{m} = \frac{\dot{W}}{\frac{\dot{W}_{3-4}}{\dot{m}} + \frac{\dot{W}_{5-6}}{\dot{m}}} &= \frac{300 \; kJ/s}{419.86 \; kJ/kg + 667.3 \; kJ/kg} = 0.2759 \; kg/s \\ &= \frac{300 \; kJ/s}{419.86 \; kJ/kg + [575.39] \; kJ/kg} = [0.3014] \; kg/s\end{aligned}$$

Applying the first law to the steam generator gives:

$$\begin{aligned}\frac{\dot{Q}_{in}}{\dot{m}} &= \frac{\dot{Q}_{2-3}}{\dot{m}} + \frac{\dot{Q}_{4-5}}{\dot{m}} \\ &= (h_3 - h_2) + (h_5 - h_4) \\ &= (3177.2 - 344.83) + (3263.9 - 2757.34) = 3338.93 \; kJ/kg \\ &= (3177.2 - 344.83) + ([3051.2] - 2757.34) = [3126.23] \; kJ/kg\end{aligned}$$

The addition of heat to the steam generator is:

$$\begin{aligned}\dot{Q}_{in} = \dot{m} \cdot \frac{\dot{Q}_{in}}{\dot{m}} &= 0.2759 \times 3338.93 = 921.21 \; kW \\ &= [0.3014] \times [3126.23] = [942.25] \; kW\end{aligned}$$

Part b)

The power required to operate the pump is:

$$\begin{aligned}-\dot{W}_{pump} = \dot{m} \cdot (h_1 - h_2) &= 0.2759 \cdot (338.7 - 344.83) = -1.69 \; kW \\ &= [0.3014] \cdot (338.7 - 344.83) = [-1.84] \; kW\end{aligned}$$

Part c)

The cycle's thermodynamic efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{(300 - 1.69)\ kW}{921.21\ kW} = 0.324$$
$$= \frac{(300 - [1.84])\ kW}{[942.25]\ kW} = [0.316]$$

Part d)

Reheat is primarily used to maximize the quality of the steam leaving the turbine (should be greater than 90%). A low quality of steam at the turbine outlet will result in water particles impinging upon the turbine blades resulting in cavitation which leads to lower efficiencies and excessive maintainence costs.

Reheat also can provide higher system efficiencies by increasing the mean temperature at which heat is added in the boiler thereby increasing the thermal efficiency.

Part d)

In this case $s_4 - s_6$. Therefore the quality at 6 can be calculated as

$$s_6 = (1-x) \cdot s_{s_f} + x \cdot s_{6_g}$$
$$6.5415 = (1-x) \cdot 1.08575 + x \cdot 7.60185$$

Therefore

$$x = 0.8373$$

Then

$$h_6 = (1-x) \cdot h_{6_f} + x \cdot h_{6_g}$$
$$= 0.1627 \cdot 338.7 + 0.8373 \cdot 2645.1$$
$$= 2269.85\ kJ/kg$$

Application of the first law to turbine 1 gives:

$$\frac{\dot{W}_{3-6}}{\dot{m}} = h_3 - h_6 = 3177.2 - 2269.85 = 907.35\ kJ/kg$$

and

$$\dot{W}_{3-6} = \dot{m} \times \frac{\dot{W}_{3-6}}{\dot{m}}$$
$$= .2759\ kg/s\ \times\ 907.35\ kJ/kg = 250.34\ kW$$
$$= [.3014]\ kg/s\ \times\ 907.35\ kJ/kg = [273.48]\ kW$$

Applying the first law to the steam generator gives:

$$\frac{\dot{Q}_{in}}{\dot{m}} = \frac{\dot{Q}_{2-3}}{\dot{m}}$$
$$= (h_3 - h_2)$$
$$= (3177.2 - 344.83) = 2832.37\ kJ/kg$$

The addition of heat to the steam generator is:

$$\begin{aligned}\dot{Q}_{in} = \dot{m} \cdot \frac{\dot{Q}_{in}}{\dot{m}} &= 0.2759 \times 2832.37 = 781.45\ kW \\ &= [0.3014] \times 2832.37 = [853.68]\ kW\end{aligned}$$

This cycle's thermodynamic efficiency is

$$\begin{aligned}\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} &= \frac{(250.34 - 1.69)\ kW}{781.45\ kW} = 0.3182 \\ &= \frac{([273.48] - [1.84])\ kW}{[853.68]\ kW} = [0.3182]\end{aligned}$$