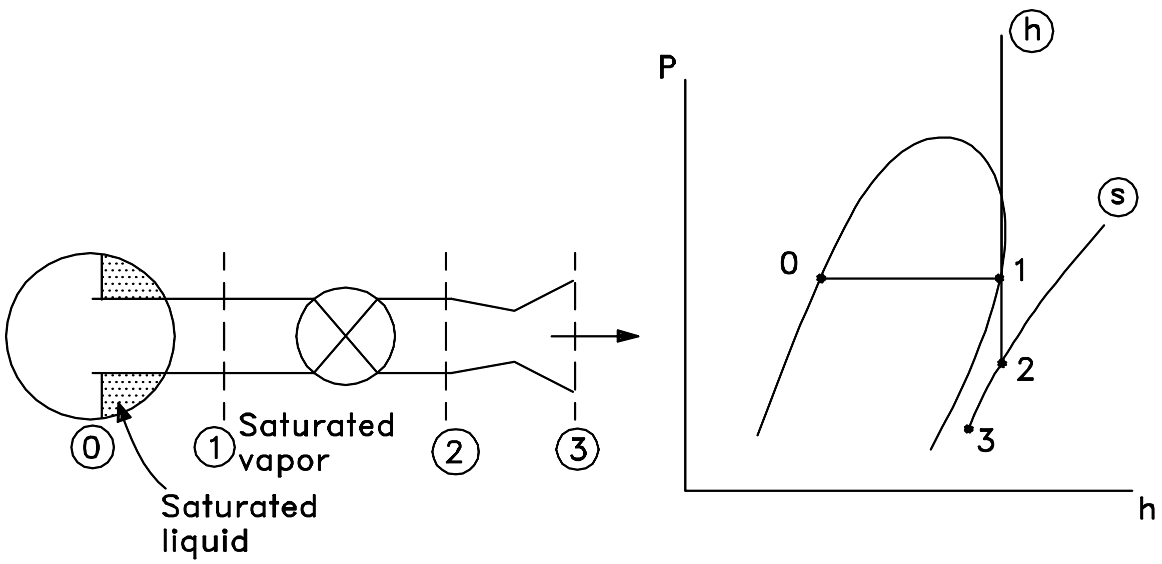
Problem: 5.43

Given: Small portable jet rocket with Freon 12
State Point 1: Saturated vapor at 80 $^\circ F$
State Point 2: $P_2 = 20\ psia$
State Point 3: $P_3 = 2\ psia \quad T_3 = -80\ ^\circ F$
Thrust: $F = \dot{m}V/g_c = 10\ lb_f$
steady flow, constant pressure in tank

Find: a) Specific impulse $SI = F/\dot{m}$
b) Volume of tank required for 1000 sec of operation
c) Total heat transfer to the Freon during that 1000 sec.



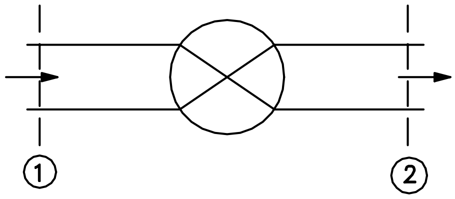
Assumptions

1. $\Delta pe_{1-3} = 0$
2. kinetic energy at $\boxed{1}$ and $\boxed{2}$ is negligible
3. $\dot{W} = 0$

Consider the valve:

$$\dot{m}_1(h_1 + \cancel{pe_1} + \cancel{ke_1}) + \cancel{\dot{Q}}^{\,0} + \cancel{\dot{W}}^{\,0} = \dot{m}_2(h_2 + \cancel{pe_2} + \cancel{ke_2}) \cancel{\frac{dE}{dt}}^{\,0}$$

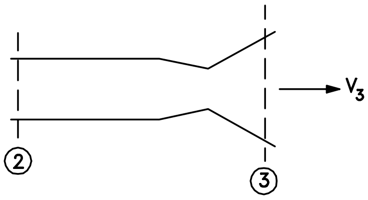
$$\dot{m}_1 = \dot{m}_2$$



Therefore

$$h_1 = h_2$$

Consider the nozzle:



$$\begin{aligned}
\dot{m}_2(h_2 + \cancel{p}e_2 + \cancel{k}e_2) + \cancel{\dot{Q}}^{\,0} + \cancel{\dot{W}}^{\,0} &= \dot{m}_3(h_3 + \cancel{p}e_3 + ke_3)\cancel{\frac{dE}{dt}}^{\,0} \\
\dot{m}_2 &= \dot{m}_3
\end{aligned}$$

Therefore

$$ke_3 = \frac{1}{2}\frac{V_3^2}{g_c} = h_2 - h_3$$

Therefore

$$V_3 = \sqrt{2g_c(h_2 - h_3)}$$

From Table B.5

$$h_1 = h_2 = h_g(\text{saturated}, 80^\circ F) = 85.282\ Btu/lbm$$

From Figure B.7

$$h_3 = h(-80^\circ F, 2\ psia) \approx 68.5\ Btu/lbm$$

$$\begin{aligned}
V_3 &= \sqrt{2\left(32.17\frac{ft \cdot lbm}{lb_f \cdot s^2}\right)(85.282 - 68.5) Btu/lbm \left(778.16\ \frac{ft \cdot lb_f}{Btu}\right)} \\
&= 916.6\ ft/s
\end{aligned}$$

$$SI = \frac{F}{\dot{m}} = \frac{\dot{m}V/g_c}{\dot{m}} = \frac{V}{g_c} = \frac{916\ ft/s}{32.17\ \dfrac{ft \cdot lbm}{lb_f \cdot s^2}} = \boxed{28.5\ \frac{lb_f \cdot s}{lbm}}$$

$$\dot{m}\frac{F}{SI} = \frac{10\ lb_f}{28.5\ lb_f \cdot s/lbm} = 0.351\ lbm/s$$

b) Size of tank required for 1000 sec. of operation.

<u>Assumptions</u>

1. Tank is initially filled with saturated liquid at 80 $^{\circ}F$
2. After 1000 sec. the tank is filled with saturated vapor (because pressure has to be held constant)

$$\begin{aligned} m_{initial} &= m_{out} + m_{final} \\ \frac{V}{v_f} &= \dot{m}\ t + \frac{V}{v_g} \Rightarrow V = \frac{\dot{m}\ t\ v_f\ v_g}{v_g - v_f} \end{aligned}$$

From Table B.5 (saturated, 80 $^{\circ}F$)

$$\begin{aligned} v_f &= 0.012277\ ft^3/lbm \\ v_g &= 0.41135\ ft^3/lbm \end{aligned}$$

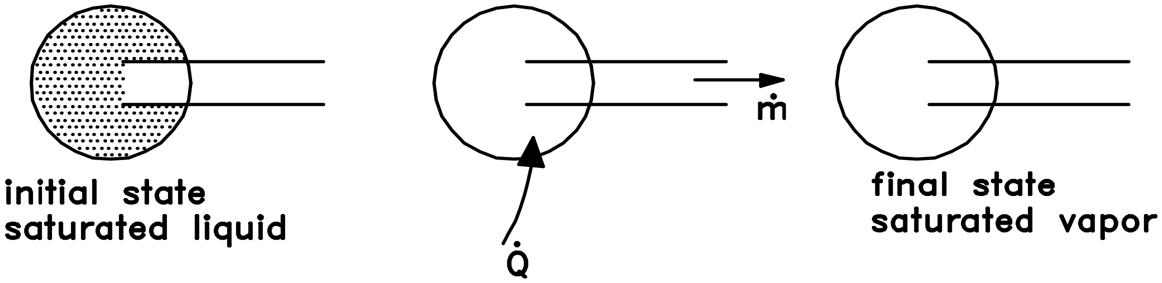
$$\begin{aligned} V &= \frac{(0.351\ lbm/s)(1000\ s)(0.012277\ ft^3/lbm)(0.41135\ ft^3/lbm)}{(0.41135 - 0.012277)\ ft^3/lbm} \\ &= \boxed{4.44\ ft^3} \end{aligned}$$

c) Heat Transfer required

First law energy balance

$$m_{initial}u_{initial} + Q + \cancel{W}^{\,0} = m_{final}u_{final} + \dot{m}\ t\ h_{out}$$

$$Q = \dot{m}_{final}u_{final} - \dot{m}_{initial}u_{initial} + \dot{m}th_{out}$$



But

$$\dot{m}\ t = m_{initial} - m_{final} \quad \text{and} \quad h_{out} = h_g$$

$$\begin{aligned} Q &= m_f u_f - m_i u_i + (m_i - m_f) h_g \\ &= m_f (u_f - h_g) + m_i (h_g - u_i) \end{aligned}$$

But

$$\begin{aligned} m_f &= \frac{V}{v_g} \\ m_i &= \frac{V}{v_f} \\ u_f &= h_g - P\ v_g \\ u_i &= h_f - P\ v_f \end{aligned}$$

$$\begin{aligned} Q &= \frac{V}{v_g}(\cancel{h_g} - P v_g - \cancel{h_g}) + \frac{V}{v_f}(h_g - h_f + P\ v_f) \\ &= \frac{V}{v_f}(h_g - h_f) = \frac{V}{v_f} h_{fg} \end{aligned}$$

From Table B.5 (saturated, 80 $^\circ F$)

$$h_{fg} = 58.917\ Btu/lbm$$

$$Q = \frac{4.44\ ft^3}{(0.012277\ ft^3/lbm)}(58.917\ Btu/lbm) = \boxed{21,313\ Btu}$$