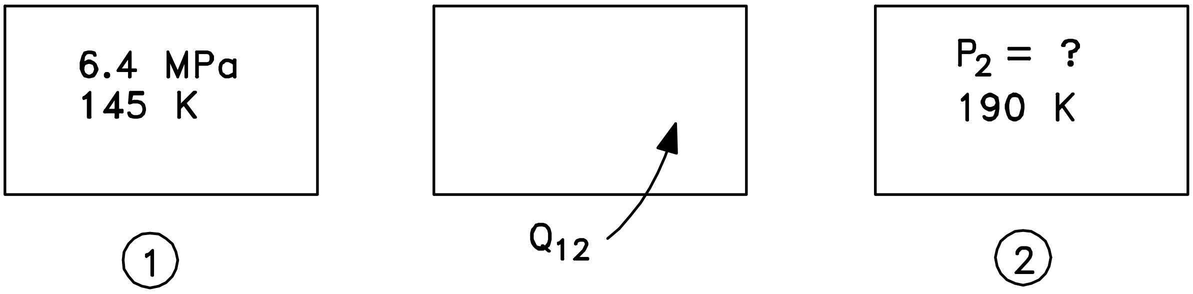
Problem: 5

Given: Nitrogen at 6.4 MPa and 145 K is contained in a tank of 0.3 m^3. Heat is transferred until the temperature is 190 K.

Find: Heat transferred and final pressure using
a) generalized charts
b) perfect gas assumption and constant specific heats



a) Energy Balance

$$E_1 + Q_{12} + \cancel{W}^{\,0}_{12} = E_2$$

$$Q_{12} = E_2 - E_1 = m(u_2 - u_1 + \Delta \cancel{ke}^{\,0} + \Delta \cancel{pe}^{\,0})$$

$$u_2 - u_1 = (h_2 - h_1) - (P_2V_2 - P_1V_1)$$

From Table B.8

$$\begin{aligned} P_{cr} &= 33.54\ atm = 3.398\ MPa \\ T_{cr} &= 126.2\ K \end{aligned}$$

$$\begin{aligned} P_1^* &= \frac{P_1}{P_{cr}} = \frac{6.4\ MPa}{3.398\ MPa} = 1.883 \\ T_1^* &= \frac{T_1}{T_{cr}} = \frac{145\ K}{126.2\ K} = 1.149 \\ T_2^* &= \frac{T_2}{T_{cr}} = \frac{190\ K}{126.2\ K} = 1.506 \end{aligned}$$

From Figure B.14a

$$Z_1(P_1^* = 1.883, T_1^* = 1.149) \approx 0.51$$

and

$$\nu_1^* = \frac{\nu_1}{\nu_{cr}} \approx 0.30$$

Since the volume of the container is constant and the mass of the nitrogen is constant

$$\nu_1 = \nu_2 \quad \text{and} \quad \nu_1^* = \nu_2^*$$

State 2 is then defined by $T_2^* = 1.506$ and $\nu_2^* = 0.30$.
From Figure B.14a

$$P_2^*(T_2^* = 1.506, \nu_2^* = 0.30) \approx 4.0$$

Therefore

$$P_2 = 4.0 P_{cr} = 4.0(3.398\ MPa) = \boxed{13.544\ MPa}$$

Let $\beta = [\hat{h}_{pg} - \hat{h}]/T_c$

From Figure B.15

$$\begin{aligned}
\beta_1(T_1^* = 1.149, P_1^* = 1.883) &\approx 5\ \frac{cal}{gmole \cdot K} \times 4.186\ \frac{kJ/lbmole \cdot K}{cal/gmole \cdot K} = 20.93\ \frac{kJ}{kgmole \cdot K} \\
\beta_2(T_2^* = 1.506, P_2^* = 4.0) &\approx 4\ \frac{cal}{gmole \cdot K} \times 4.186\ \frac{kJ/lbmole \cdot K}{cal/gmole \cdot K} = 16.744\ \frac{kJ}{kgmole \cdot K}
\end{aligned}$$

$$\begin{aligned}
h_2 - h_1 &= \left(h_{pg_2} - \frac{\beta_2 T_c}{\hat{M}}\right) - \left(h_{pg_1} - \frac{\beta_1 T_c}{\hat{M}}\right) \\
&= c_p(T_2 - T_1) + \frac{T_c}{\hat{M}}(\beta_1 - \beta_2)
\end{aligned}$$

From Table B.6b

$$\begin{aligned}
c_p &= 1.038\ kJ/kg \cdot K \\
\hat{M} &= 28.02\ kg/kgmole \\
R &= 0.296\ kJ/kg \cdot K
\end{aligned}$$

$$
\begin{aligned}
h_2 - h_1 &= \left(1.038\ \frac{kJ}{kg \cdot K}\right)(190 - 145)\ K + \frac{(126.2\ K)}{\left(28.02\ \dfrac{kg}{kgmole}\right)}(20.93 - 16.744)\ \frac{kJ}{kgmole \cdot K} \\
&= 65.563\ kJ/kg
\end{aligned}
$$

$$
\begin{aligned}
\nu_1 = \frac{Z_1 R T_1}{P_1} &= \frac{(0.51)(0.296\ kJ/kg \cdot K)(145\ K)}{6.4 \times 10^3\ kN/m^2} \\
&= 0.003420\ m^3/kg
\end{aligned}
$$

$$
m = \frac{V}{\nu_1} = \frac{0.3\ m^3}{0.003420\ m^3/kg} = 87.71\ kg
$$

$$
\begin{aligned}
Q_{12} &= m[(h_2 - h_1) - \nu(P_2 - P_1)] \\
&= (87.71\ kg)[(65.563\ kJ/kg) - (0.00342 m^3/kg)(13594 - 6400) kN/m^2] \\
&= \boxed{3592.7\ kJ}
\end{aligned}
$$

b) <u>Perfect gas with constant specific heats.</u>
Since the volume is constant

$$
\frac{P_1}{T_1} = \frac{P_2}{T_2}
$$

$$
\begin{aligned}
P_2 &= P_1 \left(\frac{T_2}{T_1}\right) = (6.4\ MPa)\left(\frac{190\ K}{145\ K}\right) \\
&= \boxed{8.386\ MPa}
\end{aligned}
$$

$$
\nu_1 = \frac{RT_1}{P_1} = \frac{(0.296\ kJ/kg \cdot K)(145\ K)}{6400\ kN/m^2)} = 0.006706\ m^3/kg
$$

$$
m = \frac{V}{\nu_1} = \frac{0.3\ m^3}{0.006706\ m^3/kg} = 44.734\ kg
$$

$$
Q_{12} = m(u_2 - u_1) = mc_v(T_2 - T_1)
$$

for a perfect gas with constant specific heats.
From Table B.6b, $c_v = 0.742\ kJ/kg \cdot K$

$$
\begin{aligned}
Q_{12} &= (44.734\ kg)(0.742\ kJ/kg \cdot K)(190 - 145)\ K \\
&= \boxed{1493.7\ kJ}
\end{aligned}
$$