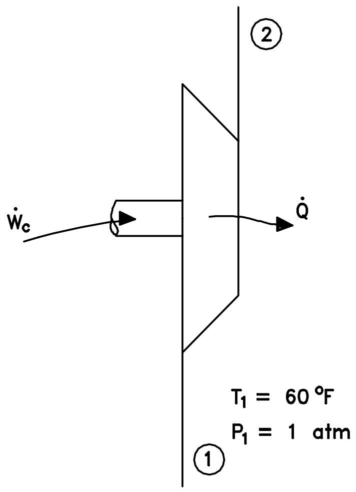
Problem: 9-53

Given: Air is compressed from 1 atm to 300 psia

Find: The isentropic and isothermal compressor efficiencies



Isentropic process

$$\dot{P}_s = \cancelto{0}{\frac{ds}{dt}}\bigg|_{C.V.} = \dot{m}\cancelto{0}{(s_2 - s_1)} + \frac{\dot{Q}_{out}}{T} \geq 0$$

Therefore

$$\frac{\dot{Q}_{out}}{T} = 0 \ \text{ for reversible case}$$

$$\dot{m}_1(h + pe + ke)_1 + \dot{W} - \cancelto{0}{\dot{Q}} = \dot{m}_2(h + pe + ke)_2 + \cancelto{0}{\frac{\partial E}{\partial t}}\bigg|_{C.V.}$$

But $\dot{m}_1 = \dot{m}_2$ for conservation of mass and $\dot{W} = \dot{m}(h_2 - h_1)$
Using the perfect gas tables

$$\begin{aligned}
h_1 &= 81.5\ Btu/lbm \\
p_{r_1} &= 40.77 \\
\frac{P_2}{P_1} &= \frac{p_{r_2}}{p_{r_1}} \rightarrow p_{r_2} = p_{r_1}\frac{P_2}{P_1} \\
p_{r_2} &= 40.77(300/14.696) = 832.27
\end{aligned}$$

at $p_{r_2} = 832.27$

$$\begin{aligned} T_2 &= 1206\ R \text{ or } 746\ ^\circ F \\ h_2 &= 250.7\ Btu/lbm \end{aligned}$$

Therefore

$$\begin{aligned} \frac{\dot{W}}{\dot{m}} &= h_2 - h_1 \\ &= 250.7 - 81.45 = 169.25\ Btu/lbm \end{aligned}$$

Using constant specific heats

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

where $k = c_p/c_v = 1.40$
Therefore

$$\begin{aligned} T_2 &= 520\left(\frac{300}{14.696}\right)^{0.4/1.4} \\ &= 1231\ R \text{ or } 771^\circ F \end{aligned}$$

and

$$\frac{\dot{W}_s}{\dot{m}} = c_p(T_2 - T_1) = 0.240(1231 - 520) = 170.6\ Btu/lbm$$

<u>Isothermal process</u>

$$\dot{P}_s = \cancelto{0}{\frac{ds}{dt}}\Bigg|_{C.V.} + \dot{m}(s_2 - s_1) + \frac{\dot{Q}_{out}}{T} \geq 0$$

Therefore

$$T(s_2 - s_1) + \frac{\dot{Q}_{out}}{\dot{m}} = 0 \quad \text{(reversible)}$$

$$\dot{m}_1(h + pe + ke)_1 + \dot{W} - \dot{Q} = \dot{m}_2(h + pe + ke)_2 + \cancelto{0}{\frac{\partial E}{\partial t}}\Bigg|_{C.V.}$$

$$\frac{\dot{W}}{\dot{m}} = \frac{\dot{Q}}{\dot{m}} + \cancelto{0}{(h_2 - h_1)}$$

Gibbs equations for a perfect gas

$$ds = c_p(T)\frac{dT}{T} - R\frac{dP}{P}$$

$$\begin{aligned}
s_2 - s_1 &= \int_{T_1}^{T_2} \cancelto{0}{c_p(T)} \frac{dT}{T} - R\ln\frac{P_2}{P_1} \\
&= 0.0685\ln(300/14.696) \\
&= 0.20661\ Btu/lbm \cdot R
\end{aligned}$$

Therefore

$$\begin{aligned}
\frac{\dot{W}_t}{\dot{m}} &= T(s_2 - s_1) \\
&= 520 \times 0.20661 \\
&= 107.4\ Btu/lbm
\end{aligned}$$

The compressor efficiency can be thought of as the ideal input work over the actual input work. In this example the ideal work is the isentropic work in the case of the isentropic efficiency and the isothermal work in the case of the isothermal efficiency. Note: efficiency may be a misleading phrase in this case because we may obtain "efficiencies" greater than 100% because our ideal condition is not a true ideal value but one based on the constraints of the problem.
The isentropic efficiency of the isothermal compressor is

$$\eta_s = \frac{\dot{W}_s}{\dot{W}_t} = \frac{170.6}{107.4} = \boxed{1.59} \quad \Leftarrow$$

The isothermal efficiency of the isentropic compressor is

$$\eta_t = \frac{\dot{W}_t}{\dot{W}_s} = \frac{107.4}{170.6} = \boxed{0.63} \quad \Leftarrow$$